Part I.

Thermodynamics and Statistical Thermodynamics

All of chemistry, including the chemistry of biological systems, can be viewed as being based on the interrelated physical factors of energetics, structure, and dynamics. In some ways, energetics can be viewed as most fundamental, since the energetic behavior of molecules determines their structure and reactivity. Thermodynamics is the branch of physical chemistry that systematizes the energetics of chemical systems at equilibrium, in terms of macroscopic concepts such as temperature, pressure, enthalpy, entropy, free energy, etc.

Biochemists and biophysicists are most often interested in the connection between structure and function of biological molecules. The bridge between the microscopic and macroscopic levels is provided by statistical mechanics or statistical thermodynamics. In this part we examine some applications of thermodynamics to biological systems, especially solutions of biologically interesting molecules, and describe how thermodynamic variables can be measured. We then discuss some of the major sources of noncovalent interactions in biomolecular systems, and how they can be estimated for interpretation of thermodynamic results. We conclude with a discussion of the statistical thermodynamics of polymer solutions.

1 THERMODYNAMIC ANALYSIS OF BIOPOLYMER SYSTEMS

1-1   GOALS AND NATURE OF THERMODYNAMIC ANALYSIS

Molecular biophysics aims to understand the molecular basis of processes such as enzymatic conversions, ion transport, ligand binding, and conformational changes of proteins and nucleic acids, that make up the reactions occuring in biological systems. The aim of a thermodynamic analysis of a biomolecular system is to measure the changes in thermodynamic parameters such as free energy, enthalpy, entropy, and volume that occur during these processes, and to use these values to gain insight into the molecular interactions that regulate the process. Since biological systems are very complex, consisting of many complex, strongly interacting molecules, a full thermodynamic analysis would be very difficult. But useful progress can be made by some relatively simple considerations.

## 1-2   RELATION OF STANDARD FREE ENERGY CHANGE TO EQUILIBRIUM CONSTANT

A great deal of understanding can be obtained from systematic application of a few basic equations from thermodynamics. The first is the relation between equilibrium constant K and standard free energy change $G°$:

$$G° \; = \; -RT \ln K \tag{1-1}$$

which rearranges to the exponential form

$$K \; = \; e^{-\frac{G°}{RT}} \tag{1-2}$$

This form emphasizes **the importance of weak interactions**, since small changes in $G°$ imply large changes in K. This can be seen explicitly by calculating various quantities for a unimolecular isomerization $N \rightleftharpoons U$ where N might stand for the native form of a protein, and U for the unfolded form (or N for the base-paired helical form of a self-complementary single-stranded oligonucleotide, and U for the denatured or coil form)
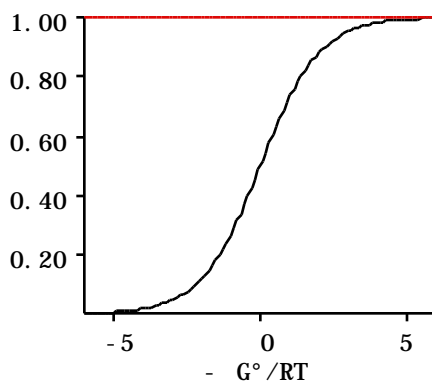
$$K \; = \; \frac{[U]}{[N]} \; = \; \frac{}{1-}$$

where   is the fraction denatured:

$$= \; \frac{[U]}{[U]+[N]} \; = \; \frac{K}{1+K}$$

As $G°$ (expressed in various units) ranges from somewhat positive to somewhat negative, we get

$G°$/kJ	$G°$/kcal	$G°/RT$	K	α
7.43	1.78	3	0.050	0.047
4.95	1.18	2	0.135	0.119
2.48	0.59	1	0.368	0.269
0.00	0.00	0	1.000	0.500
-2.48	-0.59	-1	2.718	0.731
-4.95	-1.18	-2	7.389	0.881
-7.43	-1.78	-3	20.086	0.953

Thus a change in $G°$ of 6 RT (only a few H-bonds or hydrophobic interactions) can change the fraction of the unfolded state from 5% to 95%.

It is worth noting the converse: Since $G°$ is related logarithmically to K, relatively crude measurement of K leads to quite accurate determination of $G°$.

1-3   RELATION OF FREE ENERGY TO ENTHALPY AND ENTROPY

The second major equation is

$$G° = H° - T\,S° \tag{1-3}$$

This relates the Gibbs energy change to changes in standard enthalpy $H°$ and entropy $S°$. In crude terms, $H°$ is related to the formation, breaking, or distortion of bonds; while $S°$ is related to the change in order or randomness in the system. A major effort in modern biophysics is understanding the molecular basis of the thermodynamic parameters $G°$, $H°$, and $S°$.

1-4   DETERMINATION OF EQUILIBRIUM CONSTANT FROM LINEAR PROPERTIES

Measurable properties (e.g. spectral absorption, fluorescence, circular dichroism) are often linearly proportional to concentrations. A typical relation is the Beer-Lambert law for optical density A:

$$A = \varepsilon Cl \tag{1-4}$$

where ε is molar extinction coefficient, C is molar concentration, and l is path length. A similar equation holds for each species in the system, and the total optical density is the sum over all species. For the N ⇄ U transition, for example, we have (taking $l = 1$ cm)

$$A = A_N + A_U = \varepsilon_N[N] + \varepsilon_U[U] = [\varepsilon_N(1 - \alpha) + \varepsilon_U\alpha]C_T \tag{1-5}$$

where C_T is the total protein concentration. This can be rearranged to solve for α:

$$\alpha = \frac{A - \varepsilon_N C_T}{\varepsilon_U C_T - \varepsilon_N C_T} = \frac{A - A_N}{A_U - A_N} \tag{1-6}$$

It should be emphasized that this treatment assumes that the only states contributing are N and U; that is, that this is a *two-state transition*. This is a very common assumption in biochemistry, but needs to be examined carefully in each particular case, since it may not always be true.

1-5   DETERMINING H° AND   S° FROM THE VAN'T HOFF EQUATION

If we can measure K as a function of T, we can determine the other thermodynamic parameters. The starting point is the van't Hoff equation, which may be derived as follows. We divide Eq. (1-3) by T

$$\frac{G}{T} = \frac{H}{T} - S \tag{1-7}$$

and then differentiate

$$\frac{d}{dT}\frac{G}{T} = \frac{1}{T}\frac{d}{dT}H - \frac{H}{T^2} - \frac{d}{dT}S. \tag{1-8}$$

We recall that

$$dH = Q \text{ and } dS = \frac{Q}{T} \text{ where } Q = C_P dT \tag{1-9}$$

and C_P is the heat capacity at constant pressure P so that

$$d H = C_P dT \text{ and } d S = \frac{C_P}{T} dT \tag{1-10}$$

Substituting these relations in eq. (1-8) we get

$$\frac{d}{dT}\frac{G}{T} = \frac{C_P}{T} - \frac{H}{T^2} - \frac{C_P}{T} = -\frac{H}{T^2} \tag{1-11}$$

Now using eq. (1-1), we finally obtain the van't Hoff equation which can be written in either of the two equivalent forms

$$\frac{\ln K}{T} = \frac{H°}{RT^2} \text{ or } \frac{\ln K}{(1/T)} = -\frac{H°}{R} \tag{1-12}$$

where the second form comes from noting that $\frac{}{T} = -\frac{1}{T^2 (1/T)}$. The first form of the van't Hoff equation makes it easy to see (and remember) that for an endothermic reaction ($H° > 0$), ln$K$ increases with T; while the second form suggests a linear plot of ln$K$ vs $1/T$ to obtain $H°$:
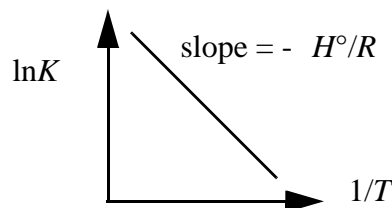
Once the standard free energy and enthalpy changes are known, the entropy change is obtained from

$$\Delta S° = \frac{(\Delta H° - \Delta G°)}{T}.$$
(1-13)

1-6   DETERMINING THERMODYNAMIC PARAMETERS FROM REACTION PROGRESS AND MOLECULARITY

In many cases, it is the fractional reaction α rather than K that is measured directly. Further, determining K from α requires a knowledge of n, the number of molecules in the reaction. That is, for proteins one can consider unfolding ($n=1$), dimerization ($n=2$), tetramerization ($n=4$), etc. For nucleic acids, one can consider melting of an intramolecular hairpin ($n=1$), melting of a duplex ($n=2$), melting of a triplex or tetraplex, etc. A general analysis for determining thermodynamic parameters from n and the T-dependence of α has been developed by Marky & Breslauer (1,2).

A distinction must be made between identical and non-identical reaction partners. Consider first identical partners, such as self-complementary oligonucleotides or protein oligomers composed of identical subunits, undergoing the reaction $n\,A \rightleftharpoons A_n$. Let α be the fraction of A chains in state A_n, where the total molar concentration of chains is C_T. The equilibrium expression is

$$K = \frac{[A_n]}{[A]^n} = \frac{(C_T/n)}{[(1-\alpha)C_T]^n} = \frac{\alpha}{nC_T^{\,n-1}(1-\alpha)^n}$$
(1-14)

If we define the "melting" temperature T_m as that at which $\alpha = 1/2$, then we see that

$$K(T_m) = \frac{1}{n(C_T/2)^{n-1}}$$
(1-15)

which allows determination of K from the total monomer concentration at the midpoint of the transition. The van't Hoff equation can be obtained in terms of α by taking logarithms of both sides of eq (1-14) and differentiating, noting that $nC_T^{\,n-1}$ is a constant:

$$\Delta H_{vH} = -R\left[\frac{\partial \ln K}{\partial(1/T)}\right] = -R\frac{\partial}{\partial(1/T)}[\ln\alpha - n\ln(1-\alpha)] = -R\left[\frac{1}{\alpha}+n\frac{1}{1-\alpha}\right]\frac{\partial\alpha}{\partial(1/T)}$$
(1-16)

ΔH_{vH} is thus related to the slope at the midpoint, where $\alpha = 1/2$, by

$$\Delta H_{vH} = -R(2+2n)\left[\frac{\partial\alpha}{\partial(1/T)}\right]_{T=T_m}$$
(1-17)

The numerical coefficient $(2 + 2n)$ is 4 for a unimolecular process, 6 for dimerization, etc.

If the reaction is between n non-identical chains, each at molar concentration C_T/n,

$$A_1 + A_2 + A_3 + \ldots A_n \ , \quad A_1 A_2 A_3 \ldots A_n \tag{1-18}$$

the equilibrium expression is

$$K = \frac{[A_1 A_2 A_3 \ldots A_n]}{[A_1][A_2][A_3]\ldots[A_n]} = \frac{(C_T/n)}{[(1-\ )(C_T/n)]^n} = \frac{}{(C_T/n)^{n-1}(1-\ )^n} \tag{1-19}$$

This differs from eq (1-14) for identical chains, but the dependence on α is the same, so eq (1-17) holds for both cases.

1-7   DIFFERENTIAL SCANNING CALORIMETRY (DSC)

Biological samples are usually small and dilute, so conventional calorimetric measurements have until recently required too much material and have lacked sensitivity. However, recent technical advances have led to the development of *microcalorimeters*, which can detect the very small amounts of heat generated or consumed by the ligand binding and conformational change reactions undergone by proteins, nucleic acids, and membranes. Binding reactions are generally studied by *isothermal titration calorimeters*, which will be described later. Here we consider *differential scanning calorimeters*, in which processes such as protein unfolding and helix-coil transitions can be studied as a function of temperature (3). A schematic diagram of a differential scanning calorimeter is shown below:
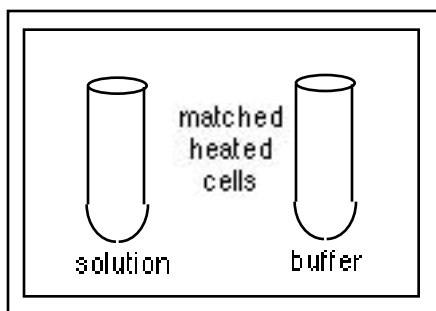
Heat is supplied at the same rate to two matched cells. The solution cell will generally absorb more heat than the buffer cell, causing a slight difference in temperature T between the two cells. A feedback loop monitoring T will supply a small amount of heat Q to the solution cell, so as to equalize the temperatures. The heat capacity Cp = Q/ T.

Determination of H and S from DSC

A schematic illustration of how DSC traces are integrated to obtain enthalpy and entropy:
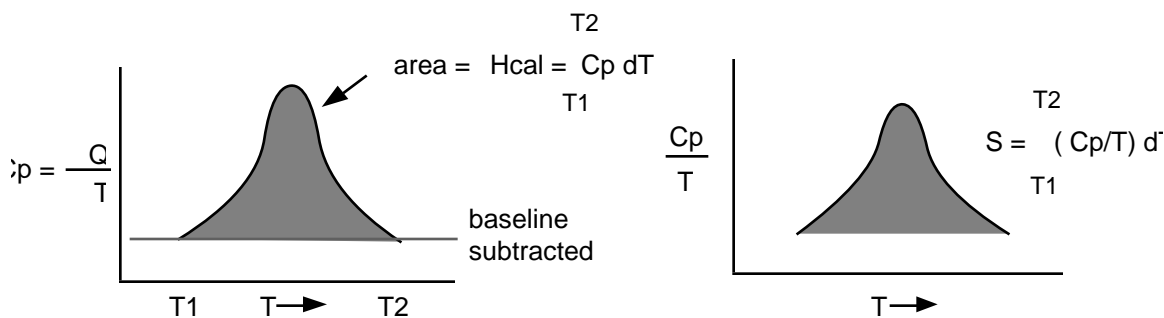
Note how the same data (C_P as a function ot T) can be used to obtain both H and S, and thereby G as well.

DSC of protein unfolding

As an example of how DSC can be used in a biologically relevant situation, consider the figure below which represents the typical thermal denaturation of a protein.

Protein unfolding

Cp

Cp of protein unfolding

Tm   T$\longrightarrow$
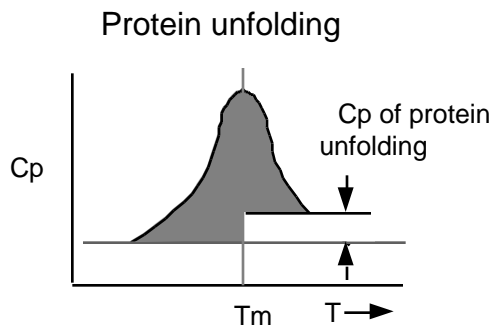
$$H^\circ{}_{cal} = \int_{T_1}^{T_2} C_p dT$$

$$S^\circ = \int_{T_1}^{T_2} \frac{C_p}{T} dT$$

$$T_M = \frac{H^\circ{}_{cal}}{S^\circ} \quad (\text{when} \quad G^\circ = 0)$$

The shaded area represents the heat input to the system to unfold the protein; the difference between the baselines at low and high T represents the difference in heat capacities between folded and unfolded forms.

1-8   RELATION BETWEEN CALORIMETRIC AND VAN'T HOFF ENTHALPIES

The two ways we have described to determine the enthalpy of a transition, H_{vH} from the temperature dependence of the equilibrium according to the van't Hoff equation, and H_{cal} from a calorimetric measurement, will not generally yield the same result. The relation between the two depends on the cooperativity of the transition.

*   If $H_{vH} = H_{cal}$, this indicates a two-state transition. However, Sturtevant (4) has noted that many reactions which are thought to be two-state do not obey this equality, for reasons currently unknown.

*   If $H_{vH} < H_{cal}$, some intermediate states are involved. One can operationally define the number n of "cooperative units" from the relation

$$\frac{H_{cal}}{H_{vH}} = n . \tag{1-20}$$

As a simple example, if a protein had n independent, identical domains, each unfolding independently, $H_{vH} = (H_{cal})/n$ since the equilibrium transition for each domain would proceed independently of the others.

*   If $H_{vH} > H_{cal}$, this generally indicates intermolecular interactions between molecules which must be overcome to get from one state to the other.

Both H_{vH} and H_{cal} can be obtained from DSC, by using eq (1-17) rewritten in the form

$$H_{vH} = RT_m^2 (2 + 2n) \left[\frac{}{T} \right]_{T = T_m} . \tag{1-21}$$

In this case   can viewed as the reaction progress variable (the fraction of reactant converted to

product), which can be evaluated from the heat capacity curve

$$(T) = \frac{1}{H_{cal}} \int_{T_1}^{T} C_P dT \quad \text{so that} \quad \left[\frac{}{T}\right]_{T = T_m} = \frac{C_{P,ex}(T_m)}{H_{cal}} \tag{1-22}$$

and

$$H_{vH} = (2n + 2)RT_m^2 \frac{C_{P,ex}(T_m)}{H_{cal}} \tag{1-23}$$

where $C_{P,ex}(T_m)$ is the excess heat capacity at the half-way point of the transition (approximately the maximum in the C_P *vs* T curve).

1-9   REFERENCES

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2 THERMODYNAMICS OF SOLUTIONS

Much of biophysical chemistry is concerned with the thermodynamic changes that occur when a residue is moved from one environment to another, *e.g.,* a nonpolar amino acid from the interior of a native protein to the aqueous solvent in the unfolded protein. Volumes and densities change when components of biomolecular solutions are mixed, and changes in concentrations and pressure cause changes of osmotic pressure and water activity. Proper treatment of these types of changes requires consideration of the thermodynamics of solutions, the topic of this chapter.

2-1   CONCENTRATION SCALES

There are many ways of describing the concentration of a solution, each useful in different situations. We will use n_i, N_i, g_i, and M_i to denote the number of moles, molecules, grams, and molecular weight of component i in the solution.

Mole fraction

The most useful concentration variable for theoretical understanding of solutions of like-sized molecules is the mole fraction X:

$$X_A = \frac{n_A}{n_A + n_B + n_C + \ldots} = \frac{n_A}{n_A + n_B} \text{ in binary solution.} \tag{2-1}$$

Molality

The molality m_B is the moles of component B per kg (1000 g) of some other component A chosen as solvent. It is the most direct for accurate preparation of solutions by weight, since it is not affected by solution volume change on mixing. The relation between solute mole fraction and molality in binary solution is

$$X_B = \frac{m_B}{\frac{1000}{M_A} + m_B} \qquad \frac{m_B M_A}{1000} \text{ in dilute solutions.} \tag{2-2}$$

Molarity

The molarity C_B is the number of moles of B per liter of solution. This is experimentally convenient if solutions are prepared by volume; and statistical thermodynamic theories are also usually developed for a defined volume. If the solution density (g/mL or kg/L) $= \rho$, the relation between mole fraction and molarity in a binary solution is

$$X_B = \frac{C_B}{\frac{1000 \ - C_B M_B}{M_A} + C_B} \qquad \frac{C_B M_A}{1000 \ _A} \text{ for dilute solutions} \tag{2-3}$$

Weight concentration

The weight concentration c_B is the grams of B per liter of solution (or mg/mL in common biochemical usage). It is useful when you don't know the molecular weight, and you make up

solutions by volume rather than by weight. Of course, $c_B = C_B M_B$. Some variants are g/mL = $c_B/1000$ (for calculations in cgs units), g/100 g solution = weight percent, and g/dL = $c_B/10$ weight percent in water

Volume fraction

The volume fraction is often used for polymer solutions, where the solute (polymer, component 2) is much larger than the solvent (component 1). If each segment of the polymer is the same size as a solvent model (commonly assumed in lattice models, though not generally true), and there are segments per polymer,

$$\phi_1 = \frac{n_1}{n_1 + \sigma n_2} \quad \text{and} \quad \phi_2 = \frac{\sigma n_2}{n_1 + \sigma n_2}. \tag{2-4}$$

2-2   EXTENSIVE AND INTENSIVE QUANTITIES

Thermodynamic description of a system involves *extensive* quantities (state functions), which are proportional to the size of the system; and *intensive* quantities, which are not. The usual extensive quantities are volume V, internal energy E, enthalpy H, Gibbs (free) energy G, entropy S, and Helmholz energy A, and the number of moles of each component. In membrane or bilayer systems the surface area is also an extensive quantity.

The usual intensive quantities are pressure P, temperature T, partial molar quantities, and applied fields (electric, magnetic, centrifugal, etc.).

2-3   PARTIAL MOLAR QUANTITIES

Because of interactions between components, the value of an extensive quantity in a solution is generally not equal to the sum of the values for the pure components. To deal with this situation, we use partial molar quantities.

For example, the partial molar volume $\overline{V}_A$ of component A is the change in volume of a solution upon adding one mole of A to a volume of solution large enough that the concentration is not changed appreciably, at constant T and P.

$$\overline{V}_A = \left(\frac{\partial V}{\partial n_A} \right)_{T, P, n_B}. \tag{2-5}$$

Adding dn_A moles of A and dn_B moles of B to a solution produces a volume change

$$dV = \left(\frac{\partial V}{\partial n_A} \right)_{T, P, n_B} dn_A + \left(\frac{\partial V}{\partial n_B} \right)_{T, P, n_A} dn_B = \overline{V}_A dn_A + \overline{V}_B dn_B. \tag{2-6}$$

This expression can be integrated, increasing the volume while maintaining the relative composition constant, so $\overline{V}_A$ and $\overline{V}_B$ don't change:

$$V = n_A \overline{V}_A + n_B \overline{V}_B \tag{2-7}$$

(rather than the sum $n_A V_A^\circ + n_B V_B^\circ$ for the pure components).

Gibbs-Duhem equation

The total differential of eq (2-7) is

$$dV = \bar{V}_A dn_A + n_A d\bar{V}_A + \bar{V}_B dn_B + n_B d\bar{V}_B. \tag{2-8}$$

Comparison with eq (2-6) yields one form of the Gibbs-Duhem equation:

$$n_A d\bar{V}_A + n_B d\bar{V}_B = 0 \text{ or } d\bar{V}_A = -\frac{n_B}{n_A} d\bar{V}_B. \tag{2-9}$$

Generalizing from a binary solution to an *n*-component system, this states that one of the partial molar quantities can be obtained from the *n* concentrations and (*n*-1) other partial molar quantities.

Partial molar quantities corresponding to other extensive variables

The same relations hold for the other extensive state variables, *e.g.*:

$$\bar{S}_A = \left.\frac{S}{n_A}\right|_{T, P, n_B}, \quad \bar{H}_A = \left.\frac{H}{n_A}\right|_{T, P, n_B}, \quad \bar{G}_A = \left.\frac{G}{n_A}\right|_{T, P, n_B} = \mu_A. \tag{2-10}$$

The last of these equations defines the chemical potential μ_A, which we will use extensively later.

Maxwell equations

For one-component thermodynamics one has the relations

$$\left.\frac{G}{P}\right|_T = V; \quad \left.\frac{G}{dT}\right|_P = -S; \quad \left.\frac{H}{T}\right|_P = C_P; \text{ etc} \tag{2-11}$$

Differentiating again with respect to the number of moles of component *i*, one gets the basic thermodynamic relations for solutions

$$\left.\frac{\bar{G}_i}{P}\right|_T = \left.\frac{\mu_i}{P}\right|_T = \bar{V}_i; \quad \left.\frac{\mu_i}{T}\right|_P = -\bar{S}_i; \quad \frac{\bar{H}_i}{T} = \bar{C}_{Pi}, \tag{2-12}$$

and so forth. These cross-derivative relations are examples of *Maxwell relations*.

2-4   EXPERIMENTAL DETERMINATION OF PARTIAL MOLAR QUANTITIES

We will take partial molar volume as an example; other partial molar quantities are calculated exactly the same way. If *B* is water and *A* is solute, $\bar{V}_A$ is the slope when the volume of solution is plotted against the *molality* of *A* (at constant 1 kg of *B*). The direct numerical differentiation of *V* with respect to m_A is not very accurate. A better way is to calculate $\bar{V}_A$ as some average *V* plus a correction term. One standard way is the "method of intercepts".

Define the molar volume of solution $\overline{V}$ as the volume per total number of moles of all components:

$$\overline{V} = \frac{V}{n_A + n_B} \qquad V = \overline{V}(n_A + n_B).$$ (2-13)

Thus

$$\overline{V}_A = \left. \frac{V}{n_A} \right|_{n_B} = \overline{V} + \left. \frac{\overline{V}}{n_A} \right|_{n_B} (n_A + n_B).$$ (2-14)

By the chain rule,

$$\left. \frac{\overline{V}}{n_A} \right|_{n_B} = \frac{\overline{V}}{X_B} \left. \frac{X_B}{n_A} \right|_{n_B} = - \frac{\overline{V}}{X_B} \frac{n_B}{(n_A + n_B)^2}$$ (2-15)

Combination of eqs (2-14) and (2-15) yields

$$\overline{V} = \overline{V}_A + X_B \frac{\overline{V}}{X_B}$$ (2-16)

Thus if a plot is constructed of $\overline{V}$ vs X_B, and a line is drawn tangent to the curve at any point, the intercept at $X_B = 0$ is $\overline{V}_A$, and the intercept at $X_B = 1$ is $\overline{V}_B$. If $\overline{V}_A$ is known as a function of concentration, $\overline{V}_B$ can also be calculated by numerical integration of the Gibbs-Duhem equation:

$$d\overline{V}_B = - \frac{n_A}{n_B} d\overline{V}_A = - \frac{X_A}{X_B} d\overline{V}_A .$$ (2-17)

Note that the partial molar volume (and other partial molar quantities) are not constants, but will be functions of concentration. In the dilute solutions most often studied in biochemistry, $\overline{V}_A$ of the solvent is often well approximated by the value for pure solvent, $V_A°$; while $\overline{V}_B$ of the solute has its limiting, infinite dilution value which may be quite different from that for pure B.

2-5   CHEMICAL POTENTIAL

The chemical potential

$$\mu_i = \overline{G}_i = \left. \frac{G}{n_i} \right|_{T, P, n_{j \neq i}}$$ (2-18)

is the key quantity in solution thermodynamics. One learns in basic physical chemistry that if a component is in equilibrium between two phases   and   at the same T and P, $\mu_i = \mu_i$.

Derivatives of μ_i with respect to T and P give

$$\frac{\mu_i}{P}_{T, n_{j \ i}} = \overline{V}_{i'} \left[\frac{(\mu_i/T)}{(1/T)} \right]_{P, n_{j \ i}} = \overline{H}_{i'} \quad \frac{\mu_i}{T}_{P, n_{j \ i}} = -\overline{S}_i \qquad (2\text{-}19)$$

2-6   THERMODYNAMICS OF MIXING

When pure components are mixed to form a solution, the change in a thermodynamic quantity such as the enthalpy is called the enthalpy of mixing, H_{mix}. It is the difference between the enthalpy of the solution and the pure components:

$$H_{mix} = H_{solution} - \quad n_i H_i^\circ = \quad n_i \overline{H}_i - \quad n_i H_i^\circ = \quad n_i(\overline{H}_i - H_i^\circ). \qquad (2\text{-}20)$$

Similar equations hold for the volume of mixing V_{mix}, the entropy of mixing S_{mix}, and so on. Note that because $\overline{G}_i = \mu_i$, the Gibbs energy of mixing can be written

$$G_{mix} = \quad n_i(\mu_i - \mu_i^\circ). \qquad (2\text{-}21)$$

2-7   IDEAL SOLUTIONS

An ideal solution is one in which all molecules have the same intermolecular forces and volumes, so that $H_{mix} = 0$ and $V_{mix} = 0$. Then the molecules in the solution can mix totally randomly, and all the thermodynamic properties of the solution, relative to the pure components, can be calculated from the entropy of mixing.

Entropy of mixing of an ideal solution

Consider the bringing together of N_1 molecules of 1 and N_2 molecules moles of 2 from the pure liquids to form a mixture with mole fraction $X_1 = N_1/(N_1 + N_2)$ and $X_2 = N_2/(N_1 + N_2)$. Use the Boltzmann equation

$$S = k_B \ln \qquad (2\text{-}22)$$

and a lattice model (not required, but simple) to calculate S for the pure components and the mixture.   is the number of *distinguishable* configurations of the system, and k_B is the Boltzmann constant. Since all the molecules in the pure components are identical, there is only one distinguishable way to arrange them, so   = 0 and $S_1^\circ = S_2^\circ = 0$.

For the mixture, let $N_0 = N_1 + N_2$ be the total number of lattice sites. Imagine placing the 1 molecules first: There are N_0 sites on which to place the first, $(N_0 - 1)$ for the second, $(N_0 - 2)$ for the third, and $(N_0 - N_1 + 1)$ for the N_1th. The total number of ways to place the type 1 molecules is the product of all of these: $N_0(N_0 - 1)(N_0 - 2) \ldots (N_0 - N_1 + 1)$. Since all molecules of type 1 are identical, the number of *distinguishable* ways to place them on the lattice is

$$\Omega = \frac{N_0(N_0 - 1)(N_0 - 2)\ldots(N_0 - N_1 + 1)}{N_1!} = \frac{N_0!}{N_1!N_2!} \tag{2-23}$$

(The molecules of type 2 are then added to all empty sites, but there is only one distinguishable way to do this.) To evaluate $S_{solution}$, we take the logarithm of Ω, using Stirling's approximation

$$\ln(N!) \approx N\ln N - N \tag{2-24}$$

which is very accurate for large N. This yields (remember $N_0 = N_1 + N_2$)

$$\ln\Omega = (N_0\ln N_0 - N_0) - (N_1\ln N_1 - N_1) - (N_2\ln N_2 - N_2) \tag{2-25}$$

$$= -N_1\ln\frac{N_1}{N_1 + N_2} - N_2\ln\frac{N_2}{N_1 + N_2}$$

or

$$S_{solution} = -N_A k_B(n_1\ln X_1 + n_2\ln X_2) = -R(n_1\ln X_1 + n_2\ln X_2) \tag{2-26}$$

where N_A is the Avogadro number and R is the gas constant. Then

$$S_{mix} = S_{solution} - S_1{}^\circ - S_2{}^\circ = -R(n_1\ln X_1 + n_2\ln X_2) \tag{2-27}$$

for an ideal solution. If the solution has c components, this can readily be generalized to

$$S_{mix} = S_{solution} = -R\sum_{i=1}^{c} n_i\ln X_i \tag{2-28}$$

Differentiation of $S_{solution}$ with respect to n_i yields

$$\bar{S}_i = -R\ln X_i \tag{2-29}$$

If there are different interactions between different types of molecular pairs, or if they occupy different volumes, then mixing will not be totally random (there will be clustering) and the entropy of mixing will be different than the ideal value. This is the case in most biologically relevant systems.

Chemical potential in ideal solutions

Since $\mu_i = \bar{H}_i - T\bar{S}_i$, one can write for ideal solutions

$$\mu_i = \mu_i{}^\circ(T, P) + RT\ln X_i \tag{2-30}$$

where the standard chemical potential $\mu_i°(T, P)$ refers to the state in which $X_i = 1$. Expressions other than eq (2-30) are frequently written, since in dilute solution X_i, C_i, c_i, and m_i are all proportional to each other if i is solute. Thus for example

$$\mu_i = \mu_i°(T, P) + RT \ln C_i \tag{2-31}$$

where the standard chemical potential in the reference state of 1 mol/L differs from the standard chemical potential in the reference state of $X_i = 1$ by a constant. Since we are generally concerned with differences in chemical potential during reactions or transfer processes, $\mu_i°(T, P)$ usually cancels out.

2-8   ACTIVITIES AND ACTIVITY COEFFICIENTS IN REAL SOLUTIONS

In real solutions the molecules are generally different in intermolecular forces and volumes, leading to non-zero H_{mix} and V_{mix}. This nonideal behavior can be subsumed in an *activity coefficient* which is a function of concentrations as well as T and P, and the concentration is generalized to an *activity*. The activity coefficient   1, and the activity approaches the concentration, as the solution approaches the standard state.

$$\mu_i = \mu_i°(T, P) + RT \ln a_i \tag{2-32}$$

where

$$a_i = _iX_i \text{ or } a_i = y_i C_i \tag{2-33}$$

depending on concentration scale. If component i is solvent, the usual standard state is pure solvent, so $_i$ 1 as X_i 1. If component i is solute, the properties of the standard state are usually those at infinite dilution. If one is using mole fraction as concentration variable, the standard state is then $X_B = 1$, pure solute but with the properties of solute completely surrounded by solvent molecules.

2-9   CHEMICAL POTENTIAL AND VIRIAL EXPANSION

Another way to describe solution nonideality is through a concentration expansion. Even for an ideal solution, it may be necessary to carry higher terms in the expansion:

$$\mu_1 - \mu_1° = RT \ln X_1 = RT \ln(1 - X_2) = -RT\left[X_2 + \frac{1}{2}X_2^2 + \dots \right] \tag{2-34}$$

In dilute solutions one has $n_2 \ll n_1$, so one can express the solute concetration in molar or molal units rather than mole fractions as

$$X_2 \quad \frac{n_2}{n_1} = C_2V_1° = \frac{c_2V_1°}{M_2} \tag{2-35}$$

where $V_1{}^\circ$ is the molar volume (liters/mole) of pure solvent. Then one has the *virial expansion* in powers of c_2

$$\mu_1 - \mu_1{}^\circ = -RTV_1{}^\circ \frac{c_2}{M_2}\left[1 + \frac{V_1{}^\circ}{2M_2}c_2 + \ldots\right] \tag{2-36}$$

The generalization of this to nonideal solutions is

$$\mu_1 - \mu_1{}^\circ = -RTV_1{}^\circ \frac{c_2}{M_2}(1 + Bc_2 + Cc_2^2 + \ldots) \tag{2-37}$$

where B is the second virial coefficient, C is the third, etc. B characterizes nonideal interactions between pairs of molecules, C characterizes 3-body effects, etc.

2-10 EFFECTS OF HYDROSTATIC PRESSURE

Biological reactions that involve changes in the exposure of charged or nonpolar groups often proceed with a change in volume, ΔV. If high pressure P (hundreds or thousands of atmospheres are readily attained) is imposed on a solution, sizeable changes in the position of equilibrium can occur (1). The equation governing this is simply that for *P-V* work:

$$\Delta G = P\Delta V \quad \text{or} \quad K(P) = K(1\text{ atm})e^{-P\Delta V/RT} \tag{2-38}$$

2-11 OSMOTIC PRESSURE AND ITS RELATION TO CHEMICAL POTENTIAL

Osmotic pressure is a common way to measure the chemical potential of polymer solutions. If polymer in solvent is separated by pure solvent by a semipermeable membrane, the polymer lowers the chemical potential of the solvent. A pressure applied to the polymer compartment raises μ_1 until equilibrium is reestablished and the solvent chemical potential is the same on both sides of the membrane. The excess pressure above atmospheric is called the osmotic pressure Π.
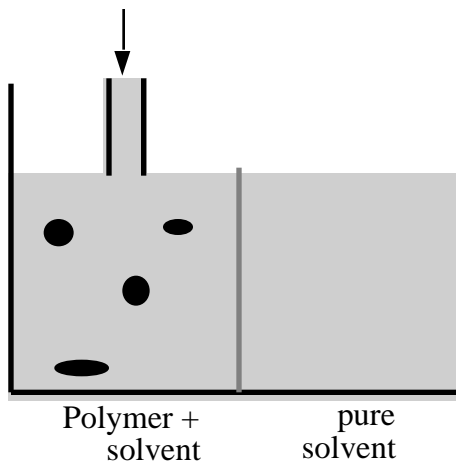
From analogy with the increase in Gibbs energy with pressure, $dG = VdP$, a term $\overline{V}_1\Pi$ must be added to μ_1 to account for the excess pressure. This increase in μ_1 just compensates the decrease in μ_1 due to the presence of the solute, given by the right hand side of eq (2-37). Thus we find

$$\Pi = \frac{RTc_2}{M_2}(1 + Bc_2 + Cc_2^2 + \ldots) \tag{2-39}$$

The value of Π/c_2 extrapolated to $c_2 = 0$ is RT/M_2, which shows how osmotic pressure can be used to determine molecular weight.

Polymer + solvent   pure solvent

Osmotic pressure is sometimes expressed in *osmolality*. Using the infinite dilution limiting forms $= RTc_2/M_2 = RTC_2$, and expressing R as 0.082 L-atm/mol-K and C_2 in mol/kg H$_2$0, we find that 1 osmol = 24 atm at 20 C.

2-12   OSMOTIC STRESS AND WATER ACTIVITY

Osmotic pressure as described above is an effect of dissolved solute on the activity of water. The equivalent of osmotic pressure can be exerted without a semipermeable membrane, if a solute added to an aqueous solution has no significant effect other than lowering the activity of water. Such solutes are called *osmolytes*. According to a useful review by Parsegian *et al.* (2), they include "several different polyethylene glycols, from M_r 100 to 20,000, methylated PEG, polyvinylpyrrolidone, several dextrans, stachyose, sucrose, glucose, trehalose, sorbitol, glycerol, betaine, glycine, and even NaCl." Osmotic pressures, expressed as osmolalities, are conveniently measured by vapor pressure osmometry (Wescor) as well as by freezing point depression and other techniques. A database of osmotic pressures for many osmolytes is maintained on two WWW sites: http://aqueous.labs.brocku.ca/osfile.html and http://abulafia.mgsl.dcrt.nih.gov/start.html.

The important aspect of these molecules is that they strongly lower the activity of water (generally by being strongly hydrated, thus effectively "removing" water from solution) while not interacting significantly with the biopolymers or ionic constituents of interest. The relation between water activity and osmotic pressure, from previous equations in this section, is simply

$$\mu_w - \mu_w{}^\circ \ = \ RT \ln a_w \ = \ -\overline{V}_w \quad . \tag{2-40}$$

There is a great deal of current research on the effect of osmolytes on a wide range of biochemical processes, arising from the realization that water is an important participant in most such reactions, along with protons, ions, and small ligands (2,3). We shall discuss some of the effects of water activity in later sections.

It no longer suffices to assert that since water is about 55 *M*, it is present in unchanging excess. It can been cogently argued that the osmotic pressure of solutions should be routinely measured along with pH, temperature, pressure, and ionic composition.

2-13   REFERENCES

1. Royer, C. (1995). Application of pressure to biochemical equilibria: The other thermodynamic variable. Meth. Enzymol. 259: 357-377.

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3 ESTIMATING MOLECULAR CONTRIBUTIONS TO THERMODYNAMIC PROPERTIES

Although thermodynamic information is useful in its own right for analysis of biochemical systems, much of the activity in this area is devoted to developing molecular interpretations of thermodynamic data. This will enable, it is hoped, a direct link between molecular structure and functional properties. Because biopolymers and aqueous solutions are so complex, the goal of adequate molecular understanding is far from realization. However, the attempts thus far have given useful insight.

3-1 INTERNAL ENERGY TERMS FOR MOLECULAR DYNAMICS CALCULATIONS

There are at least two main approaches to understanding the molecular basis of energetics of biomolecular systems. The most direct and fundamental is to express interactions between all pairs of atoms in terms familiar from small molecules: bond stretching, bending, and twisting; Lennard-Jones interactions, coulombic interactions, and hydrogen bonding. Direct quantum-mechanical calculation of these interactions is currently out of the question, so parameters (force fields) are adjusted for good agreement with experiments on small molecules containing similar atomic groupings and with an increasing body of quantum mechanical results. A typical force field used in molecular dynamics computer programs such as AMBER and CHARMM has the following internal energy (potential energy) terms:

:   (3-1)

$$V_{total} = \tfrac{1}{2}\sum_{bonds} K_r(r - r_{eq})^2 + \tfrac{1}{2}\sum_{angles} K_\theta(\theta - \theta_{eq})^2 + \tfrac{1}{2}\sum_{torsions} V_n\left[1 + \cos(n\phi - \gamma)\right]$$

$$+ \sum_{i=2}^{N}\sum_{j=1}^{i-1}\left[\frac{A_{ij}}{R_{ij}^{12}} - \frac{B_{ij}}{R_{ij}^{6}}\right] \qquad + \qquad \frac{q_i q}{\varepsilon R_{ij}} + \sum_{i=2}^{N}\sum_{j=i}^{i-1}\left[\frac{C_{ij}}{R_{ij}^{12}} - \frac{D_{ij}}{R_{ij}^{10}}\right]$$

Van der Waals   Coulombic   H-bonding
(Lennard-Jones)

The coefficients in these equations are adjusted empirically. In current work, agreement with experiment is (in selected cases) within a few kcal, which is excellent considering that the beginning and end states of a transition each have energies of thousands of kcal. However, since 1.4 kcal of free energy can displace the equilibrium by a factor of ten, even this excellent agreement is not generally adequate for reliable prediction.

3-2 ADDITIVE APPROACH TO NONCOVALENT INTERACTIONS

Molecular dynamics calculations for proteins and nucleic acids are extremely demanding, requiring hundreds of hours on the fastest supercomputers. Therefore, many approximate estimates of enthalpy and free changes in biochemical reactions are made by ignoring bonded interactions (stretching, bending, torsion) and using typical values for the noncovalent interactions which are thought to dominate energy and entropy changes in biochemical reactions. Approximate values are given in the table on the next page, along with dependence of some of the

interactions on interatomic distance r, charge q, dipole moment μ, angle between dipoles, polarizability , and dielectric constant .

TYPICAL MAGNITUDES OF NONCOVALENT INTERACTIONS

Type	Equation	Approx kcal/mol
Ion-ion	$q_1 q_2 / \varepsilon r$	14 ($\varepsilon = 8$, $r = 3$ Å)
Ion-dipole	$q_1 \mu_2 f(\ )/\varepsilon r^2$	-2 to +2 ($\mu = 2$ debye)
Dipole-dipole	$\mu_1 \mu_2 f(\ )/\varepsilon r^3$	-0.5 to 0.5
Ion-induced dipole	$q_1^2 \alpha_2 / 2 \varepsilon^2 r^4$	0.06 ($\alpha = 2 \times 10^{-24}$ cm^3)
Dispersion, and stacking of aromatic ring systems	$3 h \nu_0 \alpha^2 / 4 r^6$	0 to 10
H-bonding		-1.3 ± 0.6 per H-bond[1]
Hydrophobic		-1.3 ± 0.5 per CH$_2$[2]
Hydration force		-0.1 to 0.1 per residue

Several things are worth noting about this table. First, the approximate energy values vary widely, certainly enough to seriously affect the estimation of an equilibrium constant. Second, the last three interactions do not have equations, since they are not readily reducible to simple forms. (Hydrogen bonding may obey an interaction equation as in eq (3-1), but this is in a vacuum, while the value given in the table is pertinent to aqueous solution where water competes for H-bonds.) Third, the hydrophobic and hydration forces are largely entropic rather than enthalpic; these "collective interactions" will be discussed in more detail below.

3-3   ENTROPY CHANGES IN BIOCHEMICAL REACTIONS

The position of chemical equilibrium is determined by the free energy change which contains both enthalpic and entropic contributions: $\Delta G° = \Delta H° - T \Delta S°$. Entropy is usually just as important as enthalpy, but is often harder to understand and calculate adequately.

Translational entropy

In a bimolecular reaction, such as ligand binding to a macromolecule or dimerization of a protein, the individual translational entropy of the two partners is lost and converted to the translational entropy of the complex. Even for such an apparently simple and fundamental process, there is active disagreement (3-5) about how to calculate the translational entropy change ΔS_t in aqueous solution.

According to one point of view (3,4) the solvent, being composed of indistinguishable molecules, plays no essential role in the translational motion of solute molecules. Thus one can use the Sackur-Tetrode equation for the translational entropy of an ideal monatomic gas:

$$S_t(m) \;=\; N k_B \ln\left[\left(\frac{2\pi m k_B T}{h^2}\right)^{3/2} \frac{V e^{5/2}}{N}\right] \tag{3-2}$$

for N molecules of mass m in volume V, where e is the base of natural logarithms and h is Planck's constant. (This equation is in cgs units. To convert to molar units with standard state 1 mol/L, use $N = N_A$, molecular mass $m = M/N_A$, $V = 1000$ mL, and note that $N_A k_B = R$, the gas constant.)

In forming a bimolecular complex between reactants of mass m_1 and m_2,

$$S_t = S_t(m_1 + m_2) - S_t(m_1) - S_t(m_2). \tag{3-3}$$

According to the other point of view (5), the proper entropy change is that calculated using the ideal solution entropy of mixing expression, eq (2-28), since the entropy change in solution is considered to be due to the various distinguishable arrangements of solvent and solute molecules. For the general reaction

$$iA + jB + \ldots + kC, \quad P \tag{3-4}$$

with X = equilibrium mole fraction of product P, manipulation of eq (2-28) gives

$$S_t = R(i + j + \ldots + k - 1)\ln X + R(i\ln i + j\ln j + \ldots + k\ln k) \tag{3-5}$$

If the standard state is 1 molar in water, then $X = 1/55.5$ and $R\ln X = -8$ cal mol^{-1} K^{-1}. These two treatments lead to quite different numerical results, so which one is chosen has considerable practical importance. Although there are good scientists on both sides of this arguments, I will place my bets on the Sackur-Tetrode treatment.

Rotational entropy

If a dimer is formed from two monomers, the rotational entropy of the monomers is lost, and that of the dimer is gained. This is sometimes estimated from the statistical mechanical expression for the rotational entropy of nonlinear molecules

$$S_r = \frac{3}{2}Nk_B\ln \frac{8\ ^2Ik_BT}{h^2} \tag{3-6}$$

where I is an average moment of inertia mr^2 where r is a typical dimension of the molecule. The rotational entropy change is only infrequently taken into account in calculating the thermodynamics of complex formation.

Conformational entropy of backbone and side chains

When a protein folded in its native conformation, or a helical polypeptide, is converted to an unfolded state, the peptide backbone gains conformational entropy due to the increased number of rotational isomeric states accessible to each backbone unit. The entropy increase can be estimated from the Boltzmann equation if it is assumed that each of n backbone bonds has　rotational isomeric states in the unfolded conformation, but only one rotational state in the folded conformation. Then

$$S_{conf} = S_{unfolded} - S_{folded} = k_B\ln\ _{unfolded} = k_B\ln\ ^n = nk_B\ln \tag{3-7}$$

since $_{folded} = 1$. For example, each peptide group in the polypeptide backbone has two rotatable bonds (the　and　angles), each of which has approximately 3 rotational isomeric states. Thus if n_p is the number of amino acids in the polypeptide, $n = 2n_p$ and　3.

Displacement entropy of ions and water

The solvent plays a major role in biopolymer reactions. In solutions of highly charged DNA and RNA, cationic counterions are clustered near the negatively charged surface. The local ionic concentration may reach several molar, even though the bulk salt concentration is millimolar or lower. This locally high concentration of counterions is entropically unfavorable. A process which releases counterions to bulk solution, such as the binding of a basic protein or cationic ligand, results in an increase in entropy. The effect can be quite large, and many DNA-ligand complexes are stabilized more by entropy than enthalpy. This is discussed in more detail in the chapter on Polyelectrolytes.

Likewise, modulation of water "structure" by nonpolar or charged surfaces or ions changes the entropy of the water and is a major determinant of the equilibrium of biomolecular processes. The most prominent manifestation of this effect, the hydrophobic effect, is discussed in the next section. The effect of water structure near polar surfaces, leading to "hydration forces", is discussed in Section 3-5. We note here, in addition, that "chaotropic" or "structure-breaking" anions such as glutamate markedly increase the entropy of water and can have major effects on protein and nucleic acid equilibria.

3-4   TRANSFER THERMODYNAMICS AND HYDROPHOBIC EFFECT

Hydrophobicity

"Hydrophobic substances are defined as substances that are readily soluble in many nonpolar solvents, but only sparingly soluble in water, distinct from substances that have generally low solubility in all solvents because they form solids with strong intermolecular cohesion." (6, p.1). The hydrophobic effect is important in protein folding, and in stabilization of membranes and micelles.

The standard explanation of the hydrophobic effect is that $S > 0$ due to release of structured water from contact with nonpolar surfaces.   H is thought to be small and negligible. This explanation arises from measurement of transfer thermodynamics near room temperature, which we consider in the next subsection. However, as we shall see later, recent experiments show that this explanation needs substantial revision.

Unitary and cratic entropy

Consider the following transfer processes for a hydrocarbon (HC) to water:
1. HC in nonpolar solvent (mole fraction X)   HC in water (mole fraction X)
2. HC as pure liquid   HC in water (mole fraction X)

In (1) the starting and final mole fractions are the same, so no correction is required. In (2), the observed entropy of transfer S_{trans} is the sum of the ideal (cratic) entropy of mixing calculated from eq (2-27) and the nonideal (unitary) entropy of transfer S_u:

$$S_{trans} = S_u - R \ln X . \tag{3-8}$$

S_u contains the molecular interactions between HC and water that lead to the special properties of hydrophobicity. This division of entropy change into unitary and cratic parts was first made for hydrophobic interactions by Kauzmann (7). However, as noted above, it has been recently challenged (3).

Solubility of hydrocarbons in water

Small hydrocarbons serve as models for the nonpolar amino acid side chains of proteins. Transfer free energies, entropies, and enthalpies are actually generally measured from the solubilities of hydrocarbons. We write for the chemical potential of HC in water at mole fraction $X_{HC,w}$

$$\mu_{HC,w} = \mu°_{HC,w} + RT \ln X_{HC,w} + RT \ln f_{HC,w} \qquad (3\text{-}9)$$

where f is an activity coefficient. Likewise, for HC in pure hydrocarbon,

$$\mu_{HC} = \mu°_{HC} + RT \ln X_{HC} + RT \ln f_{HC} . \qquad (3\text{-}10)$$

At equilibrium, $\mu_{HC,w} = \mu_{HC}$, and $\ln X_{HC} = 0$ for pure HC (since $X_{HC} = 1$). Also, the activity coefficient f_{HC} 1 by definition for pure HC, and $f_{HC,w}$ 1 for a dilute solution of HC in water. Then

$$\mu°_{HC,w} - \mu°_{HC} = -RT \ln X_{HC,w} = \overline{G}°_{tr} \qquad (3\text{-}11)$$

is the free energy of transfer from HC to water. Some typical results are given in the table.

TRANSFER DATA FOR SMALL MOLECULES AT 25 C
from Kauzmann (1959); reproduced from Cantor & Schimmel, vol 1, page 285

Transfer Process	H kcal/mol	S_u cal/mol K	G_u kcal/mol
CH_4 in benzene CH_4 in H_2O	-2.8	-18	+2.6
CH_4 in ether CH_4 in H_2O	-2.4	-19	+3.3
CH_4 in CCl_4 CH_4 in H_2O	-2.5	-18	+2.9
C_2H_6 in benzene C_2H_6 in H_2O	-2.2	-20	+3.8
C_2H_6 in CCl_4 C_2H_6 in H_2O	-1.7	-18	+3.7
Liquid C_3H_8 C_3H_8 in H_2O	-1.8	-23	+5.05
Liquid $n\text{-}C_4H_{10}$ $n\text{-}C_4H_{10}$ in H_2O	-1.0	-23	+5.85
Liquid C_6H_6 C_6H_6 in H_2O (at 18°C)	0	-14	+4.07

The solubilities of liquid hydrocarbons in water are small. The table shows that this is not due to an unfavorable enthalpy of transfer.   H is generally slightly favorable (< 0) showing that interactions between HCs and water are not disfavored by intermolecular forces. The striking generalization is that S_u is strongly negative. This has been interpreted as structure being imposed on water by the exposed nonpolar groups.

For n-alkanes, Tanford (6) finds a linear relation with the amount of hydrocarbon exposed to water:

$$\mu°_{HC,w} - \mu°_{HC} = 2436 + 884 n_C = 2102 n_{CH_3} + 884 n_{CH_2} \text{ cal/mol} \qquad (3\text{-}12)$$

This additive property is often assumed in calculating the free energy change resulting from exposure of nonpolar side chains of proteins.

Hydrophobicity is an effect of the anomalous heat capacity of water exposed to hydrophobic residues.

The data tabulated above were obtained at 25 C. More recent experiments over a wide range of temperature have exposed a more complex reality: H and S are both strongly T-dependent due to large, positive, constant C_p on transferring nonpolar groups from organic solvents to water.

This is evident from the two graphs below. On the left is a model system, the transfer of benzene from organic solvent to water. On the right is denaturation of the protein myoglobin, in which nonpolar amino acid residues are transferred from the nonpolar interior to the aqueous solvent.

THERMODYNAMICS OF DISSOLVING BENZENE IN WATER

THERMODYNAMICS OF DENATURATION OF Mb
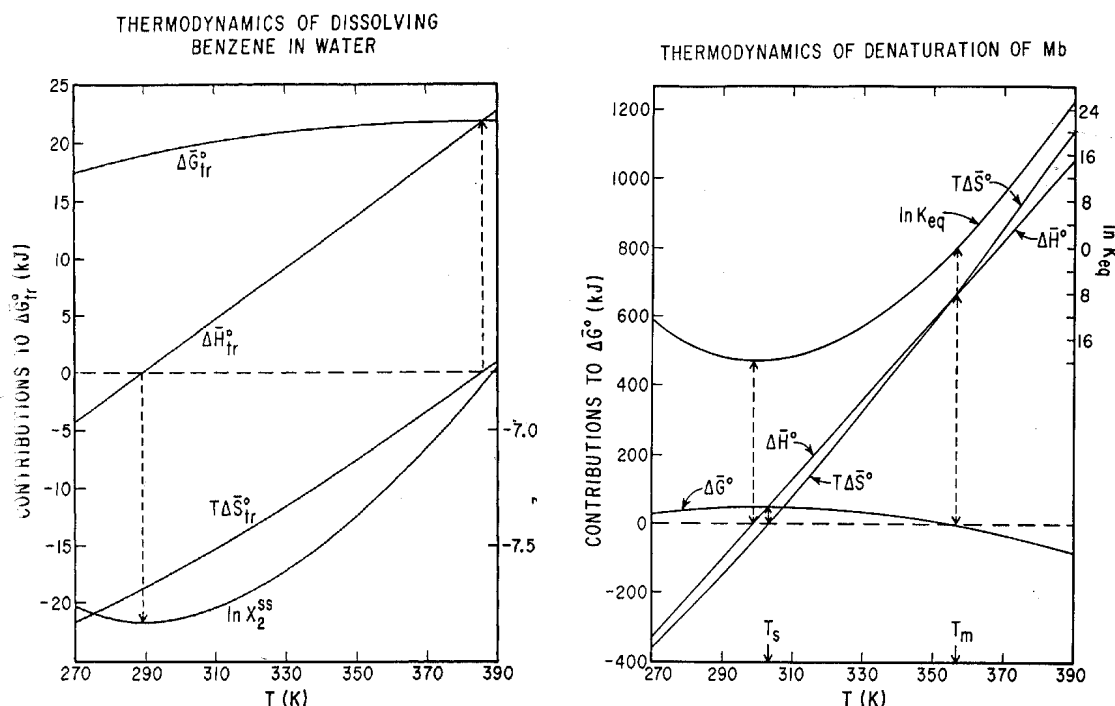
In both cases the transfer is unfavorable, but due to unfavorable entropy at low T and to unfavorable enthalpy at high T. Note that S crosses 0 at about 386 K, a general rule proposed by Baldwin (8). Thus the assertion that hydrophobic interactions are entropy-driven is true only under restricted conditions.

Integration of equations for H and S with constant C_p gives

$$H°(T_2) = H°(T_1) + C_P°(T_2 - T_1) \tag{3-13}$$

and

$$S°(T_2) = S°(T_1) + C_P° \ln(T_2/T_1) \tag{3-14}$$

which combine to give

$$G°(T_2) = G°(T_1) + C_P°[T_2 - T_1 + T_2 \ln(T_2/T_1)] - S°(T_1)(T_2 - T_1) \tag{3-15}$$

Interpretation of large C in terms of fluctuations

It is a general result from statistical mechanics that C_P is related to the fluctuations in internal energy according to the relation

$$C_P \quad C_V = \frac{1}{k_B T^2} (E - E)^2 \qquad (3\text{-}16)$$

This implies that when hydrophobic groups are transferred to water, the energy of the water can fluctuate a lot due to choices between structures imposed by hydrophobic groups.

3-5   HYDROPHOBICITY OF AMINO ACIDS

Each amino acid side chain is purported to have a separate, and additive, contribution to the total hydrophobicity of a protein in the native or unfolded state. There are many hydrophobicity scales. One of them, given in Creighton's *Proteins*, is embodied in the following MathPad program.

```
--Hydrophobicity 5/20/94

--Three-letter symbols
Ala=1;Cys=2;Asp=3;Glu=4;Phe=5;Gly=6;His=7;Ile=8;Lys=9;Leu=10;
Met=11;Asn=12;Pro=13;Gln=14;Arg=15;Ser=16;Thr=17;Val=18;Trp=19;Tyr=20

--One-letter symbols
A=1;C=2;D=3;E=4;F=5;G=6;H=7;I=8;K=9;L=10;
M=11;N=12;P=13;Q=14;R=15;S=16;T=17;V=18;W=19;Y=20

~Hydrophobicities (Creighton, 1st ed, Table 4-4, p. 142);
Relative hydrophobilies in kcal/mol~
HF={0.5,-2.8,-7.4,-9.9,2.5,0,0.5,2.5,-4.2,1.8,
1.3,-0.2,-3.3,-0.3,-11.2,-0.3,0.4,1.5,3.4,2.3}

-- hydrophobicities along sequence
hf[i]=HF[seq[i]] dim[count(seq)]
--seq={Val,Val,Val,Tyr,Phe,Gly,Gly,Gly,Lys,Lys,Lys}

seq={A,C,D,E,F,G,H,I,K,L,M,N,P,Q,R,S,T,V,W,Y}
Xmin=1; Xmax=count(seq); Ymin=-12; Ymax=5
plot hf; plot 0
count(seq):20.000
```
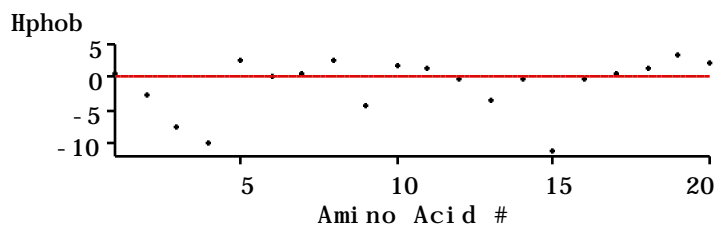
3-6   ACCESSIBLE SURFACE AREA

Agreement with experiment is more consistent if only the surface area accessible to solvent is counted in hydrophobicity calculations. Accessible surface area is defined as the area traced out by the surface of a probe sphere (usually with radius equal to that of a water molecule) on the surface of the protein (9-10):

Inaccessible

Water probe, modelled as 1.4 Å radius sphere

Surface

Interior
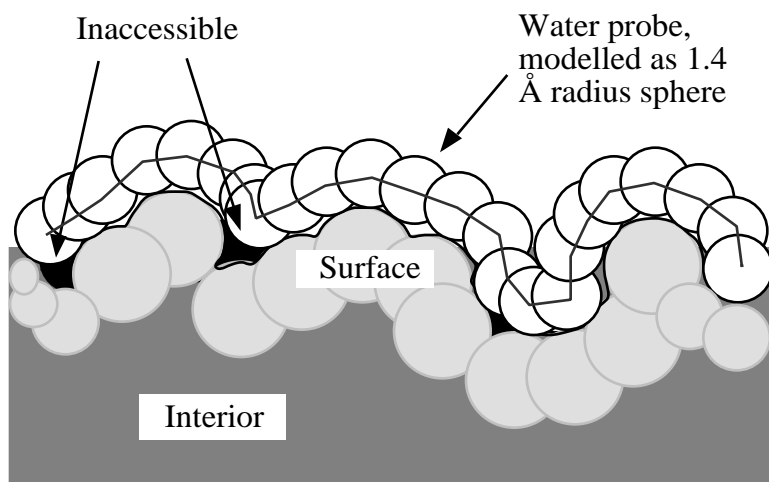
3-7   DEPENDENCE OF C_P AND S ON NON-POLAR AND POLAR SURFACE AREA

In an important series of papers, Spolar and Record and their associates (11-14) have examined a large number of model systems, and protein and protein-DNA crystal structures, and adduced regularities in the entropy and heat capacity changes associated with exposure of given areas A of both nonpolar (np) and polar (p) surfaces. Their results can be summarized by

$$C_P{}^\circ \;=\; ((0.32 \pm 0.04)\ A_{np} - (0.14 \pm 0.04)\ A_p)\ \text{cal/mol-K} \qquad (3\text{-}17)$$

and

$$S_{HE}{}^\circ \;=\; 0.32\ A_{np} \ln(T/386) \qquad (3\text{-}18)$$

where A is in Å^2 and HE stands for hydrophobic effect. Note that $S_{HE}{}^\circ = 0$ at 386 K.

3-8   HYDRATION FORCE

We are increasingly realizing that a significant component of the force between closely approaching macromolecules is due to the water at or near their surfaces (15).

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4 STATISTICAL THERMODYNAMICS OF POLYMER SOLUTIONS

Proteins and nucleic acids are polymers: large molecules built of many monomeric units covalently attached to each other. Because of their large size, polymers have unusual solution properties relative to small molecules. We discuss in this chapter how polymer size and shape affects the thermodynamic properties of polymer solutions.

4-1   EXCLUDED VOLUME AND VIRIAL EXPANSION

If the volume of the solution is V, and the volume that one molecule of solvent excludes to other molecules is u, then one can use statistical arguments to calculate the number of ways in which N_2 molecules of polymer solute can be added to the solution without overlap. This is essentially   in the Boltzmann equation $S = k_B \ln$. If there is no interaction between polymers other than excluded volume, and no interaction with solvent leading to heat effects, then it can be shown (Tanford, *Physical Chemistry of Macromolecules*, pp. 192-197) that (neglecting interactions higher than second order)
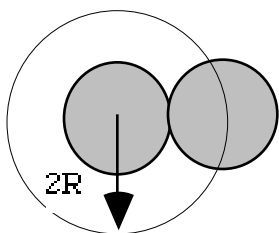
$$\mu_1 - \mu_1^{\circ} \ = \ -RTV_1^{\circ}\frac{c_2}{M_2}\left(1 + \frac{N_A u}{2M_2}c_2\right). \tag{4-1}$$

Comparing this with eq (2-37) we see that the second virial coefficient B is related to the excluded volume u by

$$B \ = \ \frac{N_A u}{2M_2} \tag{4-2}$$

Spheres

For spheres of radius R, the volume excluded by one to the centers of others is

$$u \ = \ \frac{4}{3}(2R)^3 \tag{4-3}$$

which is eight times the molecular volume, so

$$B \ = \ 4v_2 \tag{4-4}$$

where v_2 is the specific volume (cc/g) of the polymer. Higher order expansions for spheres will be considered below. It should be noted that random coil polymers are on average spherical, so their second virial coefficient may be estimated from their average radius as defined later in the chapter on polymer conformational statistics.

Rods

Independently Onsager and Zimm showed that for rods of length L and diameter d,

$$u = \frac{1}{2}\ dL^2 \qquad B \ = \ \frac{L}{d}v_2 \tag{4-5}$$
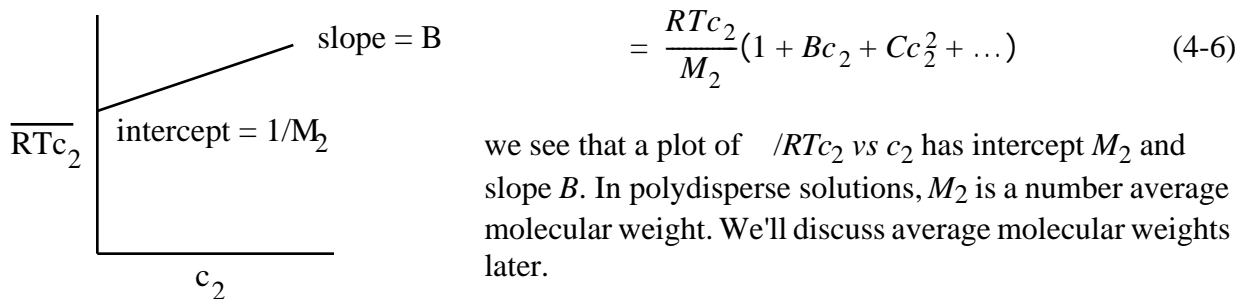
Other repulsive forces

In addition to the hard sphere and hard rod repulsive forces considered above, electrostatic and hydration forces may also contribute substantially to B. For highly charged rods, such as DNA, the effect of electrostatics is essentially to increase the diameter d by an amount several times the Debye-Hückel screening radius (1).

Attractive forces

A positive value of B implies repulsion between polymer molecules in solution. A negative value of B, on the other hand, implies attraction, and is often indicative of association or aggregation.

4-2   DETERMINATION OF MOLECULAR WEIGHT AND SECOND VIRIAL COEFFICIENT FROM OSMOTIC PRESSURE

As noted above, osmotic pressure may be used to measure the molecular weight of a dissolved solute. At the same time, from the concentration dependence of the osmotic pressure, the second virial coefficient may be determined, giving information on molecular size, shape, and interactions. Repeating eq (2-39) here for convenience;

$$\overline{\frac{\Pi}{RTc_2}} \qquad = \frac{RTc_2}{M_2}(1 + Bc_2 + Cc_2^2 + \ldots) \qquad (4\text{-}6)$$

slope = B

intercept = $1/M_2$

we see that a plot of Π/RTc_2 vs c_2 has intercept M_2 and slope B. In polydisperse solutions, M_2 is a number average molecular weight. We'll discuss average molecular weights later.

Osmotic pressure measurements with a semipermeable membrane can be made only within a restricted range of molecular sizes. If the solute is too small, it will not be possible to find a membrane that retains it but not solvent. If the solute is too large, the inverse dependence on M_2 means that Π will be too small to measure. Practically, the range of M_2 is from about 10^3 to 10^5. For smaller molecules, the equivalent colligative property, vapor pressure lowering, can be measured with a "vapor pressure osmometer".
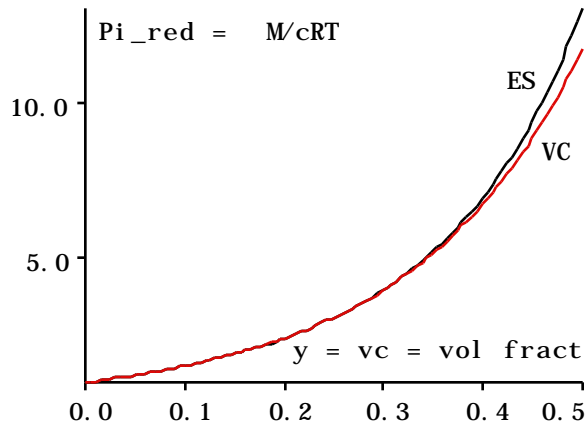
4-3   CROWDED SOLUTIONS OF RIGID PARTICLES

When solutions become very crowded, a low-order virial expansion is no longer adequate. A higher order virial expansion can be used. Terms have been worked out to B_7 for spheres (2); note that the third virial coeff is B_3, not C, in this notation:

$$\frac{M_2}{c_2 RT}\Pi = 1 + \sum_{i=2}^{7} B_i c^{i-1}. \qquad (4\text{-}7)$$

Alternatively, a closed form equation of state can be used (3):

$$\frac{M_2}{c_2 RT} = \frac{1 + y + y^2 - y^3}{(1 - y)^3} \tag{4-8}$$

where $y = c_2 v_2$ is the polymer volume fraction in solution. These two expressions agree very closely, at least up to $y = 0.5$. (ES = equation of state, VC = virial coefficient expansion)


```
Pi_red =  M/cRT

                                    ES

10.0

                                 VC


5.0


             y = vc = vol fract

0.0    0.1     0.2     0.3     0.4   0.5
```

The MathPad code for these equations is

```
--Osm press of concentrated solutions 9/26/94

--Equation of state: Carnahan, N. F. & K. E. Starling. (1969). Equation of
state for nonattracting rigid spheres, J. Chem. Phys.  51, 635-636.
--Virial coeffs: Ree, F.H. and Hoover, W.G. (1967) J. Chem. Phys. 46: 4181-
4197

--y = volume fraction of spherical polymer = vc, v = molec vol, c = molec
conc
-- Pi_red = PM/cRT, P = osmotic pressure

--Equation of state:
Pi_redES(y) = (1+y+y^2-y^3)/(1-y)^3
Xmin=0; Xmax=.5
plot Pi_redES(X)

--Virial coeffs:
B={4,10,18.36,28.24,39.5,56.4}
Pi_redVC(y)=1+sum(B[i]*y^i,i,1,count(B))
plot Pi_redVC(X)
```

In semidilute or concentrated solutions where molecular volumes overlap, the thermodynamics approximations of dilute solution theory become increasingly inadequate, as is apparent from the graph above. Biological cells are such concentrated solutions, with volume fractions of more than 0.3 occupied by macromolecules (mainly ribosomes and protein polymers). The thermodynamic consequences of highly crowded solutions have been explored by Minton (4) among others.

4-4   FLORY-HUGGINS LATTICE THEORY FOR SOLUTIONS OF FLEXIBLE POLYMERS

Up to now we have considered polymers as spheres (which may be considered models for globular proteins) or rods (models for short DNA fragments or protein polymers). These are both rigid structures, fully characterized by one or two dimensional parameters. A third major class of polymer is flexible, and has no fixed size or shape. This is a model for long DNA molecules, or unfolded proteins. It must be treated statistically.
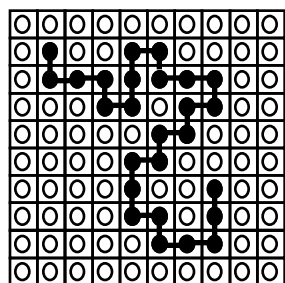
The most widely used statistical thermodynamic treatment is the Flory-Huggins lattice theory (5). In this model, each polymer segment occupies the same volume as a solvent molecule. The lattice has a coordination number (the number of neighbors of each lattice site) of z. The statistical problem of laying down N_2 chains, each containing   monomers, gives the entropy. A sample 2-dimensional configuration is shown in the figure. The result for n_1 moles of solvent and n_2 moles of polymer chains is

$$S_{mix} = -Rn_1 \ln \frac{n_1}{n_1 + n_2} - Rn_2 \ln \frac{n_2}{n_1 + n_2} = -Rn_1 \ln \phi_1 - Rn_2 \ln \phi_2 \qquad (4\text{-}9)$$

where ϕ_1 and ϕ_2 are the *volume* fractions (rather than mole fractions) of solvent and solute. Differentiating S_{mix} with respect to n_1 yields for the partial molar entropy of the solvent

$$\bar{S}_1 - S_1^\circ = -R \ln \phi_1 - R\left(1 - \frac{1}{x}\right)\phi_2 \qquad (4\text{-}10)$$

The enthalpy of mixing is calculated by assuming that the numbers of sites that have 1-1, 1-2, and 2-2 contacts are proportional to n_1^2, $n_1 n_2$, and $(x n_2)^2$, respectively; *i.e.*, that there is random mixing of solvent and polymer. The result is

$$H_{mix} = \omega n_1 \phi_2 \qquad (4\text{-}11)$$

where

$$\omega = N_A z \left(\frac{1}{2}\omega_{11} + \frac{1}{2}\omega_{22} - \omega_{12} \right) \qquad (4\text{-}12)$$

and the ω's are energies of interaction. Differentiating with respect to n_1,

$$\bar{H}_1 - H_1^\circ = -\omega \phi_2^2 \qquad (4\text{-}13)$$

Combining the entropy and enthalpy expressions, we get

$$\mu_1 - \mu_1^\circ = RT\left[\ln(1 - \phi_2) + \left(1 - \frac{1}{x}\right)\phi_2 + \frac{\omega}{RT}\phi_2^2 \right] \qquad (4\text{-}14)$$

There are two major approximations in the derivations of these equations. One is the assumption of a uniform segment density, which prohibits application to dilute solutions. The other is that the placement of segments is purely random, which cannot be the case if   is non-zero.   < 0 will favor clustering of polymer with polymer, and solvent with solvent;   > 0 will favor mixing of polymer with solvent to an extent greater than statistical. An empirical modification by Flory and Krigbaum treats   not just as an energy term, but as containing both energy and entropy contributions. They obtain an expression that sets

$$\frac{1}{2} - \frac{}{RT} \quad = \quad \left(1 - \frac{}{T}\right) \tag{4-15}$$

where   is an entropy parameter and   is a temperature, the so-called "theta temperature". When $T =$, the theory predicts that a very long polymer,   , is just on the verge of precipitating out of solution. That is,   marks a transition between "good solvent" at higher T and "poor solvent" at lower T. For a chain at the   -temperature, the second virial coefficient $B = 0$.

The lattice model of polymer solutions has been used recently, especially by Ken Dill and co-workers, to model protein folding. They use exhaustive computer enumeration to find all the chain conformations of relatively short polypeptide chains,   16 or so, and to impose a   between segments designated as hydrophobic.

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