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November 3, 2006

Financial Economics
Economics 136
Fall 2006

Economics 136: Financial Economics
Section Notes for Week 10

1 Utility and Risk Preferences

1.1 Utility

Utility $u(W)$: the measure of happiness or satisfaction achieved from a given amount of wealth.

Expected Utility $E[u(\tilde{W})]$: the measure of happiness or satisfaction expected to be achieved from a gamble (uncertain prospect) prior to the uncertainty being resolved.

1.2 Fair Gamble

A fair gamble has expected value $E[\Delta\tilde{W}] = 0$.

1.3 Concavity and Convexity

A utility function is **concave** if

- 1) it increase at a decreasing rate.
- 2) $u''(W) < 0$.
- 3) $E[u(\tilde{W})] < u(E[\tilde{W}])$ for any fair gamble $\tilde{W}$.

A utility function is **convex** if

- 1) it increases at an increasing rate.
- 2) $u''(W) > 0$.
- 3) $E[u(\tilde{W})] > u(E[\tilde{W}])$ for any fair gamble $\tilde{W}$.

1.4 Risk Aversion

A **risk averse** decision maker always turns down fair gambles and has a concave utility function.
A **risk neutral** decision maker is always indifferent to accepting fair gambles and has a linear utility function.
A **risk loving/risk seeking** decision maker always accepts fair gambles and has a convex utility function.

2 Example

Plot the following utility functions in $(W, u(W))$ space for $W \geq 0$. Also, calculate $u''(W)$ for $W \geq 0$. Do they exhibit risk averse, risk neutral, or risk loving behavior? Explain. Show whether or not someone with the given utility function and wealth $W = \$1,000$ would be willing to accept a gamble with a 50% chance of gaining \$100 and a 50% chance of losing \$100.

(a) $u(W) = 2W + 5$

ans: $u'(W) = 2 > 0$, and $u''(W) = 0$. Thus, the utility function is increasing at a constant rate. Such a utility function exhibits risk neutral behavior. Note that the stated gamble is a fair gamble since $E[\Delta\tilde{W}] = 0.5 * (100) + 0.5 * (-100) = 0$. Since

$$u(1000) = 2005 = E[u(gamble)] = 0.5(2205) + 0.5(1805) = 2005$$

someone with the given utility function would be indifferent to accepting the gamble.

(b) $u(W) = W^3$

ans: $u'(W) = 3W^2 > 0$, and $u''(W) = 6W > 0$. Thus, the utility function is increasing at an increasing rate. Such a utility function exhibits risk-loving behavior. Since

$$u(1000) = 1,000,000,000 < E[u(gamble) = 0.5(1,100)^3 + 0.5(900)^3$$

someone with the given utility function would choose to take the gamble.

(c) $u(W) = -(1/2) * e^{-2W}$

ans: $u'(W) = e^{-2W} > 0$, and $u''(W) = -2 * e{-}2W < 0$. Thus, the utility function increases at a decreasing rate. Such a utility function exhibits risk averse behavior. Since

$$u(1000) = -0.5 * e^{-2000} > E[u(gamble)] = -0.25 * e^{-2200} - 0.25 * e^{-1800}$$

someone with the given utility function would not be willing to accept the gamble.

(d) $u(W) = \frac{W^{1-\gamma}}{1-\gamma}$ where $\gamma > 1$

ans: Note that this utility function is only defined for $W > 0$. Also note that the plot of this utility function depends on the exact value of γ. For all $\gamma > 1$, $u'(W) = \frac{1}{W^{\gamma}} > 0$ and $u''(W) = \frac{-\gamma}{W^{\gamma+1}} < 0$. Thus, this utility function is increasing at a decreasing rate. Such a utility function exhibits risk averse behavior.

$$u(1000) = \frac{1000^{1-\gamma}}{1-\gamma} > E[u(gamble)] = 0.5 * \frac{1100^{1-\gamma}}{1-\gamma} + 0.5 * \frac{900^{1-\gamma}}{1-\gamma}$$

since the utility function is concave; thus, someone with the given utility function would not be willing to accept the gamble.