

Economics 136: Financial Economics
Section Notes for Week 12

1 The Capital Asset Pricing Model (CAPM)

The capital asset pricing model (CAPM) assumes

- all investors are price takers
- all investors care about returns measured over one period
- all assets are traded
- investors can borrow or lend at a given risk-free rate
- investors pay no taxes or transactions costs
- all investors are mean-variance optimizers
- all investors perceive the same means, variances, and covariances for returns.

Note: the **market portfolio** is the value-weighted portfolio of all traded assets in the economy.

If the CAPM assumptions are true, then:

The market portfolio is the tangency portfolio. Why? Because all investors are mean-variance optimizers, we know by the mutual fund theorem that they hold various combinations of the risk-free asset and the tangency portfolio. Because the market supply of risky assets must equal the market demand for assets, the market portfolio must be the tangency portfolio that investors are demanding. In other words, this result says that all investors hold combinations of the risk-free asset and the value-weighted portfolio of all traded assets.

If the CAPM assumptions are true, then:

A beta representation holds. That is, for any asset's return R_i ,

$$E[R_i] - R_f = \beta_i(E[R_m] - R_f)$$

where R_m is the return on the market portfolio and $\beta_i = Cov(R_i, R_m)/Var(R_m)$. Why? The formal argument is long, but it is based on the idea that the market return is mean-variance efficient, so you cannot increase the expected return of the market portfolio by making an adjustment into asset i without increasing your portfolio variance. Note that the expected return-beta representation of CAPM has the nice interpretation: $E[\text{excess return}] = \text{quantity of risk} \times \text{price of risk}$.

2 Testing the CAPM

In practice the CAPM can be tested by estimating the time-series regression:

$$R_{i,t} - R_{f,t} = a_i + \beta_i(R_{m,t} - R_{f,t}) + e_{i,t}$$

Here a_i = the estimated expected excess return on asset i that is not due to its risk, which is captured by $\beta_i(R_m - R_f)$; in other words a_i = the estimated abnormal return on asset i . The CAPM predicts that $a_i = 0$ for all assets.

3 Security Market Line (SML)

Security Market Line (SML) - a graph of expected returns and betas for all assets.

4 Examples

4.1 Problem 1

Answer true or false and explain how you know.

a) In a CAPM equilibrium, no risky asset will have a lower expected return than the risk-free rate.

ans: False. A risky asset whose return is negatively correlated with the market's return will have a negative beta. According to the expected return-beta representation of CAPM, this risky asset will have a lower expected return than the risk-free rate.

b) If a portfolio has $beta = 1$, it must be perfectly correlated with the market (i.e., the correlation coefficient between the market return and the portfolio return must be one).

ans: False. $beta = 1$ implies that $Cov(R_i, R_m) = Var(R_m)$. Recall that correlation $\rho(R_i, R_m) = Cov(R_i, R_m) / (\sigma(R_i) * \sigma(R_m))$. Filling in the implication, $\rho(R_i, R_m) = \sigma(R_m) / \sigma(R_i)$. Thus, correlation is not one unless $\sigma(R_m) = \sigma(R_i)$.

c) Since investors are compensated for holding risk, two securities with the same standard deviation should have the same expected return.

ans: False. The CAPM says that two securities with the same betas should have the same expected returns. In general, having the same standard deviation does not mean that betas are equal as well. For example, if these two securities with the same standard deviation have different covariances with the market return, then they will have different betas and therefore different expected returns according to the CAPM.

d) The return of XYZ's stock is perfectly correlated with the market return. According to the CAPM the expected return of XYZ's stock is equal to the market return.

ans: False. Perfect correlation implies that $Cov(R_i, R_m) = (\sigma(R_i) * \sigma(R_m))$, which implies beta of XYZ $\beta_i = \sigma(R_i) / \sigma(R_m)$. This value is not 1 in general, so the expected return of

XYZ's stock is not always equal to the expected market return.

4.2 Problem 2

Consider the following properties of the returns of stock 1, stock 2, and of the market (m): $\sigma(R_{s1}) = 0.20$, $\sigma(R_{s2}) = 0.30$, $\sigma(R_m) = 0.15$, $\rho(R_{s1}, R_m) = 0.4$, $\rho(R_{s2}, R_m) = 0.7$, and $E[R_m] = 0.10$. Also, suppose that the risk-free rate $R_f = 0.05$. a) According to the Capital Asset Pricing Model, what should be the expected return of stock 1 and of stock 2?

ans:

$$\begin{aligned}
 E[R_{s1}] &= R_f + \beta_{s1}(E[R_m] - R_f) \\
 &= R_f + \frac{\rho(R_{s1}, R_m)\sigma(R_{s1})}{\sigma(R_m)}(E[R_m] - R_f) \\
 &= 0.05 + \frac{0.4 * 0.20}{0.15}(0.10 - 0.05) = 0.0767 = 7.67\% \\
 E[R_{s2}] &= R_f + \beta_{s2}(E[R_m] - R_f) \\
 &= R_f + \frac{\rho(R_{s2}, R_m)\sigma(R_{s2})}{\sigma(R_m)}(E[R_m] - R_f) \\
 &= 0.05 + \frac{0.7 * 0.30}{0.15}(0.10 - 0.05) = 0.12 = 12\%
 \end{aligned}$$

b) Suppose that the correlation between the return of stock 1 and the return of stock 2 is 0.5. What are the expected return and the standard deviation of the return of a portfolio that has a 40% investment in stock 1 and a 60% investment in stock 2?

ans:

$$\begin{aligned}
 E[R_p] &= 0.4 * E[R_{s1}] + 0.6 * E[R_{s2}] = 0.4 * 0.0767 + 0.6 * 0.12 = 0.1027 = 10.27\% \\
 Var(R_p) &= 0.4^2 * 0.2^2 + 0.6^2 * 0.3^2 + 2 * 0.4 * 0.6 * 0.2 * 0.3 * 0.5 = 0.0532 \\
 \sigma(R_p) &= \sqrt{Var(R_p)} = 0.2307 = 23.07\%
 \end{aligned}$$

c) Assume that the Capital Asset Pricing Model is valid. Explain why you can construct a new portfolio using the market portfolio and the risk-free asset that has the same expected return as the portfolio you considered in part b) but has the lowest standard deviation possible.

ans: Assuming the CAPM is valid means that the market portfolio is the tangency portfolio. Thus, the mutual fund theorem tells us that investing in combinations of the risk-free asset and the market portfolio are the only mean-variance efficient portfolios. This means that the constructed portfolio must have a lower standard deviation than the portfolio in part b) if they are set to have equal expected returns.