

Economics 136: Financial Economics
Section Notes for Week 14

1 Forwards and Futures

Forward Contract: the arrangement for future delivery of an asset at an agreed upon price determined today called the forward price F_0^T to be paid at time T .

Spot Price - the price for immediate delivery of an asset S_0 .

Futures Contract: a standardized forward contract that trades on organized exchanges. (Note: futures contracts are “marked to market” which affects their pricing.)

Spot-Futures Parity:

$$F_0^T = S_0(1 + \text{cost of carry})^T$$

cost of carry = storage cost + foregone interest – income from holding

This relationship is derived from a no arbitrage argument.

2 Options

European Call Option - the right, but not the obligation, to buy the underlying asset at a fixed strike price (a.k.a. exercise price, X) on a given future date of expiration (T); a European call option has a payoff of $\max(S_T - X, 0)$ where S_T is the stock price on the date of expiration.

European Put Option - the right, but not the obligation, to sell the underlying asset at a fixed strike price on a given future date of expiration; a European put option has a payoff of $\max(X - S_T, 0)$.

American Options provide the right, but not the obligation, to exercise on **OR BEFORE** the date of expiration.

Options Pricing Relationships (these will be true as long as there are no arbitrage opportunities):

1. An American option can never have a lower price than an otherwise equivalent European option ($P_t^A \geq P_t^E$, $C_t^A \geq C_t^E$).
2. An American option price can be no lower than its value if exercised immediately ($C_t^A \geq S_t - X$, $P_t^A \geq X - S_t$).
3. An American option with greater time to expiration has a higher price than an otherwise equivalent American option ($C_s^A \geq C_t^A$, $P_s^A \geq P_t^A$, where $0 \leq s < t \leq T$).

4. Call options with lower strike prices are worth more than otherwise equivalent calls.
5. The price of a call is never greater than the price of its underlying security ($C_t^A, C_t^E \leq S_t$).
6. The price of a put is never greater than the its strike price ($P_t^A, P_t^E \leq X$).

3 Examples

3.1 Problem 1

a) The spot price of XYZ today is \$50, and the annual risk-free rate of return is $R_f = 5\%$. It is known that XYZ does not pay dividends during the next year. What is the forward price of XYZ for delivery 1 year from today? ...for delivery 6 months from today? ...for delivery today?

ans: Cost of carry = $0 + 0.05 - 0 = 0.05$

ans: Forward price for delivery 1 year from today = $F_0^1 = 50(1 + 0.05)^1 = \52.50

ans: Forward price for delivery 6 months from today = $F_0^{0.5} = 50(1 + 0.05)^{0.5} = \51.23

ans: Forward price for delivery today = $F_0^0 = 50(1 + 0.05)^0 = \50.00

b) Suppose an S&P 500 index portfolio pays a dividend yield of 3% annually. The index currently is 1,000. The T-bill rate is 5%, and the S&P 500 futures price for delivery in one year is \$1,010. Is the futures mispriced? If so, how can you exploit the mispricing?

ans: Spot-futures parity says that the futures price should be $= 1000(1 + 0.05 - 0.03)^1 = \$1,020$ but it's actually \$1,010. To exploit the mispricing short sell the S&P 500 index portfolio and enter into a futures contract to purchase the S&P 500 index portfolio in one year at the price \$1,010. Take the \$1,000 you get now from the shorting and put it into T-bills. This gives you \$1,050 at the end of one year, but you have to pay out the S&P dividend to the person whose shares you borrowed in the shorting = \$30. You now have \$1,020, and you use \$1,010 to purchase the S&P as agreed upon in the futures contract. You can put the remaining \$10 in your pocket.

c) Explain how volatility of the underlying asset's price affects its futures price given its spot price, the risk-free rate, and the time to delivery. Assume the asset does not pay dividends. (Hint: Use the spot-futures parity formula). Be sure to provide the economic intuition for this result.

ans: The volatility of the underlying asset's price does not affect its futures price given its spot price, the risk-free rate, and the time to delivery. Notice that the formula for a futures price doesn't change when volatility changes (volatility isn't part of the formula at all!). The intuition for this result is that agreeing now to buy at the futures price later is equivalent

to buying now (thus subsequent prices don't matter) and holding the underlying asset until the time of delivery.

3.2 Problem 2

a) Suppose you sold a European call option on ABC with strike \$50 and expiration T. Draw the payoff of your position as a function of S_T , the price of ABC, in a payoff diagram. In the same diagram, draw the payoff of selling 2 call options. Now draw the payoff of selling ten call options. What happens to the slope of the payoff function in the range where $S_T > X$ when you sell more and more calls? Does selling more options increase or reduce risk in this example?

ans: The slope of the payoff function in the range $S_T > X$ gets steeper as you sell more and more calls. Selling more call options increases risk in this example. Notice that the stock price being one dollar above the strike price results in a \$1 loss to the one call seller, but a \$10 loss to the seller of ten calls.

b) Consider the following portfolio: buy 1 share of ABC stock, buy one European put option on ABC with strike price \$40, and sell one European call option on ABC with strike price \$60. Both options expire on date T. Draw the payoff of this portfolio on date T as a function of S_T (the price of ABC at date T). What happens with your payoff if the stock price is above \$60? or below \$40? What is the advantage of this portfolio? What is the disadvantage? Do options increase or reduce risk in this example?

ans: The payoff is always \$60 at prices above \$60 because the owner of the call will exercise her right to buy your shares for \$60. The payoff is always \$40 at prices below \$40 because you will exercise your right to sell your shares for \$40 to the person who sold you the put. The advantage of this portfolio is that your downside is limited; your payoff is \$40 at lowest. The disadvantage of this portfolio is that your upside is also limited; your payoff is \$60 at highest. Options reduce your risk in this example since your payoffs from this portfolio are less variable than the payoffs from just holding the stock.