Department of Economics
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Financial Economics
Economics 136
Fall 2006

Economics 136: Financial Economics
Section Notes for Week 15: Review Problems

1 Forwards and Futures: BKM Ch. 23 Prob. 1

Consider the futures contract written on the S&P 500 and maturing in 6 months. The interest rate is 3% per 6-month period, and the future value of dividends expected to be paid over the next 6 months is \$15. The current index level is 1,425. Assume that you can short sell the S&P index.

a) Suppose the expected rate of return on the market is 6% per 6-month period. What is the expected level of the index in 6 months?

ans:

$$\begin{aligned} E[1 + R_{S\&P}] &= \frac{E[S_T + D_T]}{S_0} \\ 1.06 &= \frac{E[S_T] + 15}{1425} \\ E[S_T] &= 1.06 * 1425 - 15 = 1495.50 \end{aligned}$$

b) What is the theoretical no-arbitrage price for a 6-month futures contract on the S&P 500 stock index?

ans:

$$F_0^T = S_0(1 + R_f)^T - E[D_T] = 1425 * (1.03)^1 - 15 = 1452.75$$

or convert the dividend to a dividend yield

$$F_0^T = S_0(1 + R_f - \frac{D_T}{S_0})^T = 1425 * (1.03 - \frac{15}{)}^{1425} - 15 = 1452.75$$

c) Suppose the futures price is 1422. Is there an arbitrage opportunity? If so, how would you exploit it?

ans: Yes there is an arbitrage opportunity. The future price being offered is lower than the future price implied by spot-future parity. This means that you can enter into a future contract to buy the index at time T at a bargain. To exploit this mispricing, you should short the index in the spot market, meaning you receive 1425 today. Invest the 1425 at the risk-free rate. You end up with 1467.75 in 6 months. Then you need to pay back 15 in missed dividends to the person you borrowed the index from to short it. This leaves you with 1452.75. Next you buy the index at the 1422 that you committed to buy at. You then return the index to the person you borrowed it from and pocket $1452.75 - 1422 = 30.75$.

2 Options: BKM Ch. 20 Prob. 16

Consider the following portfolio. You write a put with exercise price 90 and buy a put option on the same stock with the same maturity date with exercise price 95.

a) Plot the value of the portfolio at the maturity date of the options.

ans: Option portfolio payoff on the vertical axis and S_T on the horizontal axis. The portfolio pays off 5 if $0 \leq S_T \leq 90$. If the $90 \leq S_T \leq 95$ the payoff decreases linearly from 5 to 0. Finally if $S_T \geq 95$ the payoff is 0.

b) On the same graph, plot the profit of the portfolio. Which option must cost more?

ans: The graph of the profit is the same as the payoff except translated down by $P_0(X = 95) - P_0(X = 90)$. The put with the strike of 95 must cost more, otherwise there would be an arbitage opportunity. If $P_0(X = 90) \geq P_0(X = 95)$ you could form a portfolio with a payoff that is always greater than or equal to 0 and sometimes strictly greater than 0 at a 0 or negative cost today.

3 Options: BKM Ch. 20 Prob. 17

A Ford option with strike price 60 sells on one exchange for \$2. On another exchange a Ford option with the same maturity but with a strike of 62 sells for \$2. Devise a zero-net-investment arbitrage strategy that involves holding positions in these options to maturity. Draw the profit diagram at maturity for your position.

ans: The option with a strike of 60 has a payoff which is greater than or equal to the option with a strike 62, and in some states of the world, strictly greater. Buy a call with strike 60 and sell a call with strike 62. Entering into this position costs nothing today. The payoff at maturity and (since the cost of setting up the position was zero) the profit at maturity is 0 if $S_T \leq 60$, increases linearly from \$0 to \$2 for $60 \leq S_T \leq 62$, and is \$2 if $S_T \geq 62$.

4 Options: BKM Ch. 20 Prob. 19

You buy a share of stock, write a 1-year call option with $X = \$10$, and buy a 1-year put option with $X = \$10$. Your net outlay to establish the entire portfolio is \$9.50. What is the risk-free interest rate? The stock pays no dividends.

ans: Assuming these are European options, we can use put call parity. $C_0 - P_0 = S_0 - \frac{X}{1+R_F}$. The problem says $-S_0 + C_0 - P_0 = -\$9.50$. Thus

$$\begin{aligned} \frac{10}{1+R_F} &= 9.50 \\ 1+R_F &= \frac{10}{9.50} \\ R_F &= 0.0526 = 5.26\% \end{aligned}$$

5 Options: BKM Ch. 20 Prob. 23

You write a call option with $X = 50$ and buy a call option with $X = 60$. The options are on the same stock and have the same maturity date. One of the calls sells for \$3 and the other sells for \$9.

a) Draw the payoff graph for this strategy at the option maturity date.

ans: The payoff is zero if $S_T \leq 50$. Payoff decreases linearly from \$0 to -\$10 in the range $50 \leq S_T \leq 60$. The payoff is -\$10 for $S_T \geq 60$.

b) Draw the profit graph for this strategy.

ans: Same as the payoff, but shifted up by \$6 (you receive \$9 for selling $X = 50$ call and pay \$3 to buy the $X = 60$ call).

c) What is the break-even point for this strategy? Is the investor bullish or bearish on the stock.

ans: Disregarding the time-value of money (ie. if $R_f = 0$) the breakeven point would be at $S_T = \$56$. Below this point you earn a profit and above this point you lose money. The investor is betting on the stock price being low, so the investor is bearish on the stock.

6 CAPM: BKM Ch. 9 Prob. 17

Suppose the rate of return on short-term government securities (perceived to be risk-free) is about 5%. Suppose also that the expected rate of return required by the market for a portfolio with a beta of 1 is 12%. According to the CAPM (security market line):

a) What is the expected rate of return on the market portfolio?

ans: $E[R_{\beta=1}] = R_f + \beta(E[R_m] - R_f) = 5\% + 1(12\% - 5\%) = 12\%$

b) What would be the expected return on a stock with $\beta = 0$?
ans: $E[R_{\beta=0}] = R_f + \beta(E[R_m] - R_f) = 5\% + 0(12\% - 5\%) = 5\%$

c) Suppose you consider buying a share of stock at \$40. The stock is expected to pay \$3 in dividends next year and you expect to sell it then for \$41. The stock risk has been evaluated at $\beta = -0.5$. Is the stock overpriced or underpriced?

ans: $E[R_{\beta=-0.5}] = R_f + \beta(E[R_m] - R_f) = 5\% - 0.5(12\% - 5\%) = 1.5\%$ says the CAPM

$$P_0 = \frac{E[P_1] + E[D_1]}{1 + R} = \frac{41 + 3}{1 + 0.015} = \$43.35$$

So the stock is underpriced according to the CAPM.