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Financial Economics
Economics 136
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Economics 136: Financial Economics
Section Notes for Week 2
mostly adapted from Keith Gamble's Section Notes (GSI from last semester)

1 Concepts

1.1 Prices, Payoffs, and Returns

We will start by defining our notation:
P_t is the price of a security at time t.
D_{t+1} is the dividend paid by a security at time $t+1$.

The payoff of a security at time $t+1$ is equal to the price of the security P_{t+1} plus any dividends that are paid out in period $t+1$. Since prices typically jump lower after a divdend payout, it is standard practice to think of the dividend payout D_{t+1} occurring an instant before the price P_{t+1} is quoted. Formally, we say that P_{t+1} is the "ex-dividend" price of the security in period $t+1$.

We define the **net simple return** from period t to period $t+1$ as,

$$R_{t+1} = \frac{payoff_{t+1} - price_t}{price_t} = \frac{P_{t+1} + D_{t+1} - P_t}{P_t} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1$$

the **gross simple return** from period t to period $t+1$ is,

$$1 + R_{t+1} = \frac{payoff_{t+1}}{price_t} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

the **log return** is defined as the natural logarithm of the gross return,

$$r_{t+1} = \log(1 + R_{t+1})$$

Note: whenenever I write log I mean natural logarithm.

1.2 Adjusting for Inflation "Real" vs. "Nominal"

In the U.S. prices are quoted in dollar amounts. However, inflation means that what you have bought with one dollar 50 years ago is likely to cost more than one dollar today. For this reason, dollars are a somewhat arbitrary index with which to measure the returns of financial assets. It makes more sense to measure the return in terms of what goods you can actually buy with it.

1.2.1 Consumer Price Index (CPI)

The Consumer Price Index (CPI) is an index defined to reflect the price changes for a constant basket of goods. It is defined so that the growth rate of the CPI is equal to the rate of inflation. We define Π_{t+1} to be the net simple rate of inflation between period t and $t+1$.

$$1 + \Pi_{t+1} = \frac{CPI_{t+1}}{CPI_t}$$

the log inflation rate between period t and $t+1$ is defined as,

$$\pi_{t+1} = \log(1 + \Pi_{t+1}) = \log(CPI_{t+1}) - \log(CPI_t)$$

1.2.2 Real Return

The real net simple return is defined as,

$$\frac{1 + R_{t+1}}{1 + \Pi_{t+1}} - 1 = \frac{R_{t+1} - \Pi_{t+1}}{1 + \Pi_{t+1}}$$

1.3 Portfolios

A combination of several assets. For example, you could have a portfolio consisting of a shares of asset 1 and b shares of asset 2.

1.3.1 Portfolio Payoff

Assuming asset 1 has a payoff equal to the random variable $X_{1,t+1}$ and asset 2 has a payoff equal to the random variable $X_{2,t+1}$ then the payoff of the example portfolio will be,

$$aX_{1,t+1} + bX_{2,t+1}$$

1.3.2 Portfolio Return

Similarly, assuming the net simple returns of assets 1 and 2 are $R_{1,t+1}$ and $R_{2,t+1}$, respectively, then the return of the portfolio will be,

$$R_{p,t+1} = w_{1,t}R_{1,t+1} + w_{2,t}R_{2,t+1}$$

where $w_{1,t}$ and $w_{2,t}$ are the fractions of total wealth invested in each asset. Thus $w_{1,t}+w_{2,t} = 1$ and $w_{1,t} = \frac{aP_{1,t}}{aP_{1,t}+bP_{2,t}}$ and $w_{2,t} = \frac{bP_{2,t}}{aP_{1,t}+bP_{2,t}}$.

1.4 Law of One Price (LOOP)

If the law of one price holds then the price of a portfolio of assets must equal the weighted (by number of shares) sum of the prices of of the assets in the portfolio,

$$P_{p,t} = P_t(aX_{1,t+1} + bX_{2,t+1}) = aP_{1,t} + bP_{2,t}$$

1.5 Arbitrage

An arbitrage opportunity exists when you have a positive probability of making money AND no probability of losing money. Note, this is stronger than just having an investment opportunity that has a positive expected value.

1.6 Redundant Asset

An asset that has the exact same payoffs in all states as a linear combination of other assets

1.7 Replicating Portfolio

The linear combination of assets that has the exact same payoffs in all states as a redundant asset.

1.8 How to Find and Exploit Arbitrage Opportunities

1) Look for a redundant asset. If you find one, then create its replicating portfolio. If they have different prices today, then you've found an arbitrage opportunity. To exploit it, short the expensive one and buy the cheap one. You'll pocket the difference in price today and pay nothing tomorrow.

2) Look for/Create assets with the same price today. If one of the assets has a better payoff in some state and no worse payoff in any state, then short the worse one and buy the better one. You'll loose nothing today and may gain something tomorrow.

1.9 Indexes

Perhaps the most helpful way to answer questions concerning different weighting schemes for indices is to remember what's made equal in forming portfolios to match the returns of the index:
1) equally weighted - invest an equal number of dollars in each stock
2) price weighted - invest in an equal number of shares of each stock
3) value weighted - invest in an equal percentage of the each stock's market capitalization
4) free float weighted - invest in an equal percentage of each stock's free float value (mkt. cap. minus govt. holdings)

2 Problems

2.1 Prices, Payoffs, and Returns

(a) Calculate the nominal net simple return for Generico in 1980 given: stock price on January 1, 1980 was \$100; stock price on December 31, 1980 was \$107.50; the company paid a dividend of \$.50 per share on December 30, 1980.

ans: nominal net simple return = (107.50+.50)/100 - 1 = .08 or 8%

ans: note that no adjustment for inflation is necessary here to get the nominal return.

(b) Calculate the real net simple return given that the inflation rate in 1980 was 14% per year.

ans: real net simple return = 1.08/1.14 - 1 =-.0526 or -5.26%

(c) Which of these would you use to compare the 1980 performance of Generico to that of other companies? Which would you use to compare it to the company's performance in some other year, say 1990?

ans: If you are comparing the 1980 performance of Generico to that of other companies in 1980 in the same country (where the same inflation rate applies), then comparing nominal returns would give you an accurate comparison of the companies' performances. You could also compare their real returns.

ans: If you are comparing the 1980 performance to performance in some other year (when inflation would likely be different), you can only accurately compare the company's performance using real returns. Notice that in the example above the nominal return for the company 8%, which looks pretty good until you consider that inflation was 14% so the stock in the company went down in terms of its purchasing power. Note that if in 1990 inflation was 0% and the stock's nominal return was 5%, then by comparing nominal returns, you would inaccurately think that the company performed better in 1980 than 1990.

2.2 Indexes

Suppose you are constructing a portfolio of two companies. The first has 10,000 shares outstanding at a current price of $100 per share, while the second has 60,000 shares outstanding at a current price of $50 per share. You have $10,000 to invest.

a) How many shares of each company would you buy if you want to construct an equal-weighted portfolio? How many shares of each company would you buy if you want to construct a value-weighted portfolio?

ans: To construct an equal-weighted portfolio, we need to an equal number of dollars in each company. Since we have $10,000 to invest and there are two companies, we need to invest $5,000 in each company's stock. We will purchase 5,000/100 = 50 shares of the first company and 5,000/50 = 100 shares of the second company.

ans: To construct a value-weighted portfolio, we need to purchase an equal percentage of each company's market capitalization. The market capitalization of the first is 10,000*100 = 1,000,000, and the market capitalization of the second is 60,000*50=30,000*100=3,000,000. If x is the equal percentage of each company's market capitalization that we'll invest in, then 1,000,000*x+3,000,000*x=10,000. x=1/400, so we much purchase 1,000,000/400 = $2,500 of the first company's stock and 3,000,000/400 = $7,500 of the second company's stock. This translates into buying 2,500/100 = 25 shares of the first company and 7,500/50 = 15,000/100= 150 shares of the second company.

b) A year later, company 1's share price has risen to \$150 while company 2's share price is unchanged at \$50. Neither company has paid a dividend. What are the returns on the equal-weighted portfolio and the value-weighted portfolio you constructed in part a)? Explain the difference.

ans: The (net simple) return of company 1's shares is (150-100)/100 = .5, and the return of company 2's shares is 0. Now we apply the idea that portfolio return is the weighted average of its individual stock returns where the weights are the shares of wealth invested in each stock. The equal-weighted portfolio return is .5*.5+.5*0=25%. The value-weighted portfolio return is .25*.5+.75*0=12.5%. The equal-weighted portfolio has a higher return because it held a greater share of wealth in the stock with the higher return.

c) How must you trade a year later to keep the equal-weighted portfolio equal-weighted? How must you trade to keep the value-weighted portfolio value-weighted? Explain. (Note: It is sufficient to explain whether you need to buy or sell the shares of either company. You do not need to calculate the exact amounts.)

ans: To keep the equal-weighted portfolio equal-weighted you must sell some of the first company's shares and buy some more of the second company's shares. Since the first company's stock price appreciated and the second company's did not, the wealth share held in the first company's stock before rebalancing is greater than 50%.

ans: To keep the value-weighted portfolio value-weighted nothing must be done. Since the number of shares outstanding hasn't changed for either company, the relative market capitalization has changed in proportion to the change in share prices. The relative wealth share held in each company has also changed in proportion to the change in share price.

2.3 Arbitrage

For each of the following situations determine if there is an arbitrage opportunity. If so, explain the source of the arbitrage opportunity and how you would trade to exploit it. If not, explain why not.

$State$	$Asset1$
s_1	\$1,000,000
s_2	\$7,000,000
$Price$	\$0.01

ans: There is no arbitrage opportunity. Asset 1 may seem like a good investment, but that does not make it an arbitrage opportunity. An arbitrage opportunity requires no initial outlay of cash. Buying asset 1 requires giving up a penny today.

$State$	$Asset1$	$Asset2$
s_1	\$7	\$4
s_2	\$5	\$2.5
$Price$	\$6	\$3

ans: There is an arbitrage opportunity. Buying two shares of asset 2 costs 6 today and pays off either 8 or 5 tomorrow. One share of asset 1 costs 6 today but only pays off either 7 or 5 tomorrow. An investor who short sells 1 share of asset 1 and uses that money purchase 2 shares of asset 2 looses nothing today nor tomorrow, but will gain tomorrow if state 1 occurs.

$State$	$Asset1$	$Asset2$	$Asset3$
s_1	\$1	\$0	\$4
s_2	\$0	\$1	\$2
$Price$	\$0.7	\$0.2	\$3

ans: There is an arbitrage opportunity. Notice that a portfolio of 4 shares of asset 1 and 2 shares of asset 2 has exactly the same payoffs as one share of asset 3. The price of this portfolio is 3.2, but the price of asset 3 is only 3. Thus an arbitrageur could short sell one share of this portfolio and use part of the proceeds to buy one share of asset 3. This strategy costs nothing today nor tomorrow and produces .2 today.

$State$	$Asset1$	$Asset2$
s_1	\$1	\$2
s_2	\$1	\$0.5
$Price$	\$0.8	\$1

ans: There is no arbitrage opportunity. First, notice that the two assets are not redundant (thus one can't short the expensive one, buy the cheap one and pocket money today). Also notice that 5 shares of asset 1 and 4 shares of asset 2 have the same price; however, neither of these portfolios produces an unambiguously (weakly) higher payoff than the other next period.