

**Economics 136: Financial Economics**  
**Section Notes for Week 3**

## 1 AoA and LOOP

### 1.1 Relationship Between AoA and LOOP

In this class we will frequently use the law of one price (LOOP) and absence of arbitrage (AoA) interchangeably. In fact, they are slightly different. AoA is stronger. AoA implies that LOOP holds.

### 1.2 Example: LOOP Holds but Arbitrage Opportunities Exist

To show that AoA and LOOP are not equivalent, give an example of a case where LOOP holds, yet an arbitrage opportunity exists.

ans: The following example works.

<i>State</i>	<i>Asset1</i>	<i>Asset2</i>
$s_1$	1	1
$s_2$	0	1
<i>Price</i>	1	1

Since there are no redundant assets, there is no possibility for LOOP to fail, therefore LOOP holds. However, Asset 2's payoffs are as good or better than Asset 1's payoffs and in state  $s_2$  they are better. Since the prices are equal, buying Asset 2 and shorting Asset 1 is an arbitrage opportunity.

Suggest a change to prices or payoffs that would eliminate the arbitrage opportunity.

ans: Any reduction in the price of Asset 1 or increase in the price of Asset 2 will eliminate the arbitrage opportunity. Changing the payoff of Asset 2 in state  $s_2$  to zero would also result in AoA.

Suggest a change (or changes) to prices or payoffs that would cause LOOP not to hold.

ans: For there to be a possibility of LOOP not holding we must have a redundant asset. Next, we need different prices for the redundant assets. Here is an example.

<i>State</i>	<i>Asset1</i>	<i>Asset2</i>
$s_1$	0.5	1
$s_2$	0	0
<i>Price</i>	1	1

## 2 Options

### 2.1 Call Options

A **European call option** is the right but not the obligation to buy the underlying asset  $S$ , at the expiration date  $T$ , at a prespecified price  $X$  (the strike price). The payoff of a European call is thus,

$$C_T = \max\{S_T - X, 0\}$$

### 2.2 Put Options

A **European put option** is the right but not the obligation to sell the underlying asset  $S$ , at the expiration date  $T$ , at a prespecified price  $X$  (the strike price). The payoff of a European call is thus,

$$P_T = \max\{X - S_T, 0\}$$

### 2.3 Put-Call Parity

Note: we have moved from looking at models of the world where a finite number of discrete events can occur to a model where a continuum of states may occur. Specifically, the price of the underlying stock can take on any non-negative value at time  $T$ . Each possible value of  $S_T$  is a possible state of the world that could occur at time  $T$ .

Let's consider an economy with four assets: a stock  $S$ , a risk-free bond, a put option on the stock  $P$ , and a call option on the stock  $C$ . The options both have the same expiration date  $T$  and strike price  $X$ . Looking at the payoffs of the four assets reveals that one of the assets is redundant. If we assume that LOOP holds, then this redundancy implies that the price of the portfolio formed by buying the call and selling the put must be equal to the price of the portfolio formed by buying the stock and borrowing the strike price. This is because these combinations have the exact same payoffs.

Payoff of portfolio 1 (buy call, sell put)

$$C_T - P_T = \max\{S_T - X, 0\} - \max\{X - S_T, 0\} = S_T - X$$

Payoff of portfolio 2 (buy stock, borrow strike)

$$S_T - X$$

Since these payoffs are equivalent, if LOOP holds then the portfolio prices must be equal. Let  $S_t$ ,  $C_t$ , and  $P_t$  denote the price of the stock, call, and put at time  $t$ . Also, note that the price of borrowing the strike  $X$  at the risk free rate is  $\frac{X}{1+R_f}$  (where  $R_f$  is the net simple return of the risk-free bond from time  $t$  to  $T$ ).

$$C_t - P_t = S_t - \frac{X}{1 + R_f}$$

or equivalently,

$$C_t + \frac{X}{1 + R_f} = S_t + P_t$$

This is the relationship known as put-call parity. Note, if we consider “today” to be time 0 then we get the form written in lecture or section,

$$C_0 + \frac{X}{1 + R_f} = S_0 + P_0$$

(now  $R_f$  has to be the net simple return of the risk-free bond from time 0 to  $T$ ).

## 2.4 Option Example

Consider a company called GM, which has issued both stocks and bonds. The economy may be either in a recession or in a boom one year from now. You have the following economic data:

<i>State</i>	<i>GMstock</i>	<i>T – Bill</i>
$s_1$	10	5
$s_2$	2	5
<i>Price</i>	5	4

What is the (net simple) rate of return  $R_f$  on an investment that gives you a sure payment one year from now (i.e., what is the riskfree rate of return)?

ans: Notice that the T-Bill provides a sure payment one year from now; thus, its return must be the riskfree rate.  $R_f = 5/4 - 1 = 1/4$  or 25%.

What are the payoffs in the two states of the world of a European call option on GM stock that has strike price  $X = 5$  and expires one year from now?

ans: The payoff to a European call option is  $\max(S_T - X, 0) = \max(10 - 5, 0) = 5$  in  $s_1$  and  $\max(2 - 5, 0) = 0$  in  $s_2$ .

What are the payoffs in the two states of the world of a European put option on GM stock that has strike price  $X = 5$  and expires one year from now?

ans: The payoff to a European put option is  $\max(X - S_T, 0) = \max(5 - 10, 0) = 0$  in  $s_1$  and  $\max(5 - 2, 0) = 3$  in  $s_2$ .

Assuming LOOP holds, find the price of a portfolio that consists of buying the call and selling the put.

ans: To answer this question first notice that the payoffs the portfolio of buy call, sell put are the same as the payoffs of the portfolio buy stock, sell bond.

<i>State</i>	<i>GMstock</i>	<i>T - Bill</i>	<i>Call(X = 5)</i>	<i>Put(X = 5)</i>
$s_1$	10	5	5	0
$s_2$	2	5	0	3
<i>Price</i>	5	4	?	?

Since the payoffs of these portfolios are the same, LOOP implies that the prices of the portfolios must also be the same.

<i>State</i>	<i>GMstock - (T - Bill)</i>	<i>Call(X = 5) - Put(X = 5)</i>
$s_1$	5	5
$s_2$	-3	-3
<i>Price</i>	$5 - 4$	?

Therefore, the price of the portfolio consisting of buying the call and selling the put must be  $5 - 4 = 1$ . This is an example of put-call parity.

Note, in at least one of my sections (Tuesday), I said that you couldn't price the call or put individually without the price of the other. That would be true if the economy had larger number of states (say if the stock price could take on any non-negative value at time  $T$  (infinite number of states)). However, in this example there are only two states and we have two linearly independent assets (GM stock and T-Bill) which we have prices for. Using these two assets, we can construct Arrow-Debreu securities and find their prices using LOOP. Then we can price the put and call using the Arrow-Debreu securities.

To form AD1 the Arrow-Debreu security that pays 1 dollar in state 1 and 0 dollars in state 2 take  $(1/8)(1 \cdot \text{GM stock} - (2/5)\text{T-Bill})$ . The price of this asset is  $(1/8)(1 \cdot 5 - (2/5) \cdot 4) = 17/(5 \cdot 8) = S_1$ .

To form AD2 the Arrow-Debreu security that pays 1 dollar in state 2 and 0 dollars in state 1 take  $(1/8)(2 \cdot \text{T-bill} - 1 \cdot \text{GM stock})$ . The price of this asset is  $(1/8)(2 \cdot 4 - 1 \cdot 5) = 3/8 = S_2$

Then the call price is  $C_0 = 5S_1 = 17/8$ . The put price is  $P_0 = 3S_2 = 9/8$ .

Thank you to the student who pointed this out.