Department of Economics
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Financial Economics
Economics 136
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Economics 136: Financial Economics
Section Notes for Week 4

1 Arrow-Debreu Securities

An **Arrow-Debreu Security** for state i is an asset that pays off \$1 in state i and \$0 in all other states. For example if three possible states are possible a year from now then,

$State$	$AD1$	$AD2$	$AD3$
s_1	1	0	0
s_2	0	1	0
s_3	0	0	1
$Price$	S_1	S_2	S_3

AD1, AD2, and AD3 are Arrow-Debreu securities for states 1, 2, and 3 respectively.

2 Complete Markets

If an Arrow-Debreu security exists for each possible state i then we say that markets are complete. What this means is that recreate any possible payoff using a simple linear combination of Arrow-Debreu securities. After recreating the payoffs, we can price the combination using LOOP. Thus for any payoffs X_1, X_2, and X_3,

$State$	$AD1$	$AD2$	$AD3$	$AssetX$
s_1	1	0	0	X_1
s_2	0	1	0	X_2
s_3	0	0	1	X_3
$Price$	S_1	S_2	S_3	$P_X = X_1S_1 + X_2S_2 + X_3S_3$

3 Dynamic Trading

Dynamic trading refers to models of the world where information is revealed sequentially, and at each point when information is revealed securities can be traded. Think of the sequential coin toss economy from lecture. A sequence of two coin tosses occur. There are three period of time: $t = 0$ (before the first coin toss), $t = 1$ (after the first coin toss, but before the second coin toss), and $t = 2$ (after the second coin toss). If dynamic trading were allowed in this model, you could purchase an asset at $t = 0$ which pays off at $t = 1$, then invest the payoff in a different asset that pays off at $t = 2$.

4 Dynamic Completeness

Sometimes (as in the coin toss example from lecture) it is possible to have fewer assets than final states of the world but still be able to create a full set of Arrow-Debreu securities for the final states by trading after some information is revealed but before the final state of the world is revealed. In lecture, this involved trading at $t = 1$ in a way that was dependent upon the outcome of the first coin toss. When Arrow-Debreu securities can be created for all possible final states of the world through the use of dynamic trading, we say that the economy is dynamically complete.

5 Dynamic Trading Example - taken from Keith Gamble (GSI last semester)

In The Big Game the possible outcomes are as follows:

- In the first half Cal can go ahead by 10 points or Stanfurd can go ahead by 10 points.
- In the second half the change in the score can be 10 points in Cal's favor or 10 points in Stanfurd's favor.

Suppose there are bookmakers who allow you to bet at the start of the game on the half-time score.

- The cost of a bet that pays \$1 if Cal is ahead at half time is \$0.80.
- The cost of a bet that pays \$1 if Stanford is ahead at half time is \$0.20 (naturally reflecting the higher probability of Cal being ahead).

These bookmakers also operate at half time.

- The cost of a bet that pays \$1 if Cal scores more than Stanfurd in the 2nd half is \$.90.
- The cost of a bet that pays \$1 if Stanfurd scores more than Cal in the 2nd half is \$.10.

The bookies are happy to take fractional bets.

a) Draw the event tree. At each node of the tree, enter the Cal lead (the number of points that Cal is ahead, a negative number if Stanford is ahead). On each branch, write the price of a bet that pays \$1 if events unfold along that branch. If we only care about the final score, how many states of the world are there at the end of the game?

ans: drawing of the tree

b) Show that even though bookmakers do not offer bets at the start of the game on the final outcome of the game, you can still use a sequence of bets, starting at the beginning of the game, to obtain \$1 if Cal wins the game, or \$1 if Stanfurd wins, or \$1 if the game is a tie. How much does it cost to obtain each of these payments?

ans: To obtain \$1 if and only if Cal wins the game, buy 9/10th of a bet that Cal will lead after the 1st half. If Cal is ahead at the half, then invest the \$.90 winnings into 1 bet that

Cal will score more than Stanfurd in the 2nd half. This strategy pays off $1 iff Cal wins. It initially costs (9/10)*.8 = $.72.

ans: To obtain $1 iff Stanford wins, buy 1/10th of a bet that Stanfurd will lead after the 1st half. If Stanfurd is ahead at the half, then invest the $.10 winnings into a bet that Stanfurd will score more than Cal in the 2nd half. This strategy costs (1/10)*.2 = $.02

ans: To obtain $1 iff there's a tie, buy 1/10th of a bet that Cal will lead after the 1st half and 9/10th of a bet on Stanford. If Cal is ahead, invest the $.10 winnings into a 2nd half Stanfurd bet. If Stanford is ahead, invest $.90 into a 2nd half Cal bet. This strategy initially costs (1/10)*.8+(9/10)*.2 = $.26.

c) Does your answer to (b) depend on the probabilities of the various outcomes? Why or why not?

ans: When you fix the prices of the bets, the answer to b) does not depend on the probabilities. Notice that the answer to b) never directly used the probabilities of the outcomes! If the prices of the bets are affected by the probabilities, then the answer to b) will change. For example, if Cal becomes even more likely to win the game (say because half Stanford's team got the flu the day before the game), then you'd expect the market price of betting on Cal to rise. This change in the price of the bets would change the answer to b).

6 Fixed Income Example - taken from Keith Gamble (GSI last semester)

A 90-day Treasury bill sells for $9850 per $10,000 face value, while a 180-day Treasury bill sells for $9750 per $10,000 face value. Using the formulas of lecture 7,

a) Calculate the bank discount yields on the two bills.

90-day T-Bill,

$$\frac{10,000 - 9,850}{10,000} \cdot \frac{360}{90} = 0.06 = 6\%$$

180-day T-Bill,

$$\frac{10,000 - 9,750}{10,000} \cdot \frac{360}{180} = 0.05 = 5\%$$

b) Calculate the bond equivalent yields on the two bills. Explain why you get different numbers than in part a).

90-day T-Bill,

$$\frac{10,000 - 9,850}{9,850} \cdot \frac{365}{90} = 0.0618 = 6.18\%$$

180-day T-Bill,

$$\frac{10,000 - 9,750}{9,750} \cdot \frac{365}{180} = 0.052 = 5.20\%$$

You get different numbers because the formulas are different. The bond equivalent yield is a more accurate measure of the true T-bill yield because it divides by the actual price (as a return measure should) and uses 365 days (as is true for non-leap years). However, neither is the best measure of the actual return of the T-bill since neither uses compounding to annualize the yield.

c) Calculate the correct annualized return by compounding (assume that there are 365 days in the year). This is also called the effective annual yield. Explain why you get different numbers than in part b).

90-day T-Bill,

$$R^{90-day} = \frac{10,000 - 9,850}{9,850} = 0.0152$$

To correctly annualize this return,

$$\begin{aligned} 1 + R^{ann} &= (1 + R^{90-day})^{\frac{365}{90}} \\ R^{ann} &= (1 + .0152)^{\frac{365}{90}} - 1 = .0632 = 6.32\% \end{aligned}$$

180-day T-Bill,

$$R^{90-day} = \frac{10,000 - 9,750}{9,750} = 0.0256$$

To correctly annualize this return,

$$\begin{aligned} 1 + R^{ann} &= (1 + R^{90-day})^{\frac{365}{90}} \\ R^{ann} &= (1 + .0152)^{\frac{365}{90}} - 1 = .0527 = 5.27\% \end{aligned}$$