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May 10, 2007

Microeconomic Theory
Economics 101A
Spring 2007

**Economics 101A: Microeconomic Theory**
**Review for Final Exam**

# 1 Optimization

## 1.1 Concave Function

## 1.2 Convex Function

## 1.3 First Order Conditions

## 1.4 Second Order Conditions

## 1.5 Constrained Maximization

# 2 Consumer Preferences and Consumer Choice

## 2.1 Utility Function (preferences) and Indifference Curves

Utility function, sometimes referred to simply as preferences, capture how individuals trade like different amounts of goods. Some common utility functions:

- Linear $u(x_1, x_2) = \alpha x_1 + \beta x_2$
- Cobb-Douglas $u(x_1, x_2) = x_1^\alpha x_2^\beta$
- Leontief $u(x_1, x_2) = \min\{\frac{x_1}{\alpha}, \frac{x_2}{\beta}\}$

Remember, for linear and Leontief preferences demand functions, indirect utility functions, and expenditure functions may not change smoothly as prices or income changes. In these cases more thought is required then simply chugging through a LaGrangian. Be especially careful about preferences such as

- Circle $u(x_1, x_2) = x_1^2 + x_2^2$

These change smoothly, so it may appear that the LaGrangian is yielding an answer that makes sense however the tangency will give you the minimum possible utility subject to a budget constraint rather than the maximum possible utility.

## 2.2 Marginal Rate of Substitution

Tangency condition equates $MRS_{x_1,x_2}$ and ratio of prices $\frac{p_1}{p_2}$.

# 3 Indifference Curve Analysis

## 3.1 Utility Maximization Problem (UMP)

$$\max_{x_1,x_2} u(x_1,x_2) \text{ s.t. } p_1x_1 + p_2x_2 \leq I$$
$$L(x_1,x_2,\lambda) = u(x_1,x_2) - \lambda(p_1x_1 + p_2x_2 - I)$$

The solutions to the UMP are the ordinary demand functions

$$x_1(p_1,p_2,I)$$
$$x_2(p_1,p_2,I)$$

Plugging the ordinary demand functions back into the utility function gives the indirect utility function which tells us the maximum level of utility for a given set of prices and level of income.

$$V(p_1,p_2,I) = u(x_1(p_1,p_2,I), x_2(p_1,p_2,I))$$

Taking the partial derivative of the value function with respect to a price we get

$$\frac{\partial V(p_1,p_2,I)}{\partial p_1} = -\lambda x_1(p_1,p_2,I)$$

a result known as Roy's Identity.

Taking the partial derivative of the value function with respect to income gives

$$\frac{\partial V(p_1,p_2,I)}{\partial I} = \lambda$$

revealing why we interpret the LaGrange multiplier as the shadow price of the constraint. It tells us how much maximized utility would increase if we had a dollar more of income.

## 3.2 Expenditure Minimization Problem (EMP)

$$\min_{x_1,x_2} p_1x_1 + p_2x_2 \text{ s.t. } u(x_1,x_2) \geq u^0$$

The solutions to the EMP are the compensated demand functions

$$x_1^c(p_1,p_2,u^0)$$
$$x_2^c(p_1,p_2,u^0)$$

Multiplying the compensated demand functions by their respective prices and summing gives the expenditure function which tells us the minimum level of dollars that we need to spend given a set of prices to achieve a level of utility, $u^0$.

$$e(p_1,p_2,u^0) = p_1x_1^c(p_1,p_2,u^0) + p_2x_2^c(p_1,p_2,u^0)$$

Taking the partial derivative of the expenditure function we get

$$\frac{\partial e(p_1,p_2,u^0)}{\partial p_1} = x_1^c(p_1,p_2,u^0)$$

a result known as Sheperd's Lemma.

# 4 Comparative Statics

The Slutsky equation decomposes the demand response to a change in price into income and substitution effects

$$\frac{\partial x_1(p_1,p_2,I)}{\partial p_1} = \frac{\partial x_1^c(p_1,p_2,u^0)}{\partial p_1} - \frac{\partial x_1(p_1,p_2,I)}{\partial I} x_1^0$$

where $x_1^0$ is the optimal level of $x_1$ demanded before the tiny change in price occurs.

## 4.1 Insta-Slutsky Derivation

Start with the identity that at an optimum ordinary and compensated demand are equal also note that at an optimum all income is spent so $I = e(p_1, p_2, u^0)$.

$$\begin{aligned} x_1^c(p_1,p_2,u^0) &= x_1(p_1,p_2,I) \\ &= x_1(p_1,p_2,e(p_1,p_2,u^0)) \end{aligned}$$

now take the partial with respect to $p_1$ of both sides

$$\begin{aligned} \frac{\partial x_1^c(p_1,p_2,u^0)}{\partial p_1} &= \frac{\partial x_1(p_1,p_2,e(p_1,p_2,u^0))}{\partial p_1} + \frac{\partial x_1(p_1,p_2,e(p_1,p_2,u^0))}{\partial e} \cdot \frac{\partial e(p_1,p_2,u^0)}{\partial p_1} \\ &= \frac{\partial x_1(p_1,p_2,I)}{\partial p_1} + \frac{\partial x_1(p_1,p_2,I)}{\partial I} \cdot x_1^c(p_1,p_2,u^0) \\ &= \frac{\partial x_1(p_1,p_2,I)}{\partial p_1} + \frac{\partial x_1(p_1,p_2,I)}{\partial I} \cdot x_1^0 \end{aligned}$$

# 5 Market Level Supply and Demand

Often assume constant elasticities of supply and demand if we considering small changes in price or income.

Elasticity is defined as:

$$\frac{\%\Delta x}{\%\Delta p} = \frac{\frac{x'-x^0}{x^0}}{\frac{p'-p^0}{p^0}} \approx \frac{\partial x}{\partial p} \cdot \frac{p}{x} = \frac{\partial \log x}{\partial \log p}$$

Several types of elasticities

- Price elasticity of demand (or elasticity of demand for short): $\eta_{xx} = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x} \leq 0$
- Income elasticity of demand (or income elasticity): $e_x = \frac{\partial x}{\partial I} \cdot \frac{I}{x}$
- Cross price elasticity of demand (or cross price elasticity for short): $\eta_{xy} = \frac{\partial x}{\partial p_y} \cdot \frac{p_y}{x}$
- Elasticity of supply: $\sigma_x = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x} \geq 0$

## 6 Labor Supply

Similar to demand for goods, but endowment point changes. Instead of the goods $x_1, x_2$ you consume $x$, all purchased goods and $L$, leisure. The endowment point changes beacuse everyone is born with leisure time then gets to sell their time in the labor market.

## 7 Consumption and Savings

Similar to demand for goods, but instead of two different goods $x_1, x_2$ we are interested in the amount of all goods consumed in period 1, $c_1$ and the amount of all goods consumed in period 2, $c_2$. Now the endowment point can be anywhere depending upon how much income the consumer is endowed with in period 1 and 2, $y_1, y_2$, respectively.

## 8 Firm Production and Cost

Suppose a firm produces output $y$ which it sells in a competitive market at price $p$. The firm uses input factors $a, b$ which it purchases in a competitive factor market at prices, $w_a, w_b$ to produce $y$ via the production technology summarized by the production function $y = f(a, b)$.

### 8.1 Two-Step: Expenditure Min, Profit Max

#### 8.1.1 Step One

$$\min_{a,b} w_a a + w_b b \text{ s.t. } f(a, b) = y$$

Solution of this problem yields the input requirement functions (IRF) or conditional factor demand functions

$$a(w_a, w_b, y)$$
$$b(w_a, w_b, y)$$

These are analagous to the consumer's compensated demand functions. Multiplying the IRF's by the respective factor prices and summing gives the cost function (analagous to the consumer's expenditure function)

$$C(w_a, w_b, y) = w_a a(w_a, w_b, y) + w_b b(w_a, w_b, y)$$

#### 8.1.2 Step Two

$$\max_{y} py - C(w_a, w_b, y)$$

leading to the first order condition

$$p = \frac{\partial C(w_a, w_b, y)}{\partial y}$$

### 8.2 One-Step: Profit Maximization Problem

$$\max_{a,b} pf(a,b) - w_a a - w_b b$$

# 9 Supply Determination

Market supply curve equals the sum of supply curves for individual firms. Sum graphically by stacking horizontally. Remember to consider the firms decision to enter the market at all (profits $> 0$).

# 10 Consumer and Producer Surplus

- Deadweight Loss: Can approximate using formula for area of a triangle and elasticities of supply and demand.
- Change in Consumer Surplus: $\Delta CS = \int_{p_x^0}^{p_x'} x(p_x, p_y, I)dp_x$
- Compensating Variation: Amount of cash you need to get back to your old utility level under the new prices. $CV =$
- Equivalent Variation: $EV = e(p_x', p_y^0, u^0) - e(p_x^0, p_y^0, u^0)$

If ordinary and compensated demand curves are equal everywhere then there are no income effects. In this case only, willingness to pay (CV or EV) is equal to the change in consumer surplus ($\Delta CS$).

# 11 Monopoly

# 12 Duopoly

Cournot Equilibrium (which is a Nash Equilibrium).

# 13 Game Theory

Nash Equilibrium

# 14 Choice Under Uncertainty

# 15 Moral Hazard

Individual can vary the level of effort they put forth to prevent an accident. Under full coverage they don't make enough effort.

# 16 Adverse Selection

Two or more different types of people: low risk and risk risk. Similar to job market signalling model. Need a separating equilibrium for firms to stay in business.

# 17 Auctions

Auction types. Optimal bidding functions. Winner's curse.

# 18 Finance

## 18.1 CAPM

## 18.2 Efficient Market Hypothesis

# 19 Public Goods and Externalities

# 20 Common Property and Congestion

# 21 Empirical Methods