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Microeconomic Theory
Economics 101A
Spring 2007

Economics 101A: Microeconomic Theory
Solutions To Practice Midterm II

1 Part I

1.1 Question 1

Uncertain. This depends upon prefererences and whether the consumer is a net borrower or saver. First, savings is defined as $A - C_1$ where A is the amount of wealth or income that the agent is endowed with in the first period and C_1 is the level of goods consumed in the first period.

For a net saver, the income effect will cause the consumer to increase C_1 and C_2, which means that savings decreases. However, the substitution effect will cause the consumer to decrease C_1 which means savings increases. Thus the total effect is ambiguous for a net saver.

For a net borrower, the income effect will cause the consumer to decrease C_1 and C_2, which means that savings increases. The substitution effect will also cause the consumer to decrease C_1 which means that savings increases. Thus the total effect is that savings increase for a net borrower.

Remember that the size of substitution effects depend upon the curvature of indifference curves. Consumers with Leontief preferences (the limiting case of extremely curved indifference curves) will have no substitution effect, while consumers with very flat indifference curves will have a large substitution effect.

1.2 Question 2

Uncertain. If the firm was not producing because the orignal price was not high enough for the firm to overcome its fixed costs and earn a positive profit and the rise in price still does not allow the firm to earn a positive profit then the firm will continue to stay out of the market and thus the rise in price will not cause the firm to increase its supply of output. Once the price is high enough that the firm will earn positive profits then firms will increase output in response to a rise in price if there marginal cost schedule is not vertical in that region.

1.3 Question 3

False. Assuming the BART operates as a monopoly, and assuming that the marginal cost of an additional passenger is close to zero, then it will be optimal for the BART to set price such that quantity of riders is limited to the point where marginal cost equals marginal revenue (where marginal revenue equals zero in this case). This may very well be a quantity of riders that is less than capacity.

2 Part II

2.1 Question 4

The change in consumer surplus understates the increase in income needed to allow the consumer to get back to his/her original utility. To see this, note that the definition of compensating variation is "the amount of cash you'd need to get back to u^0 at the new prices" or,

$$\begin{aligned} CV &= e(p_x', p_y^0, u^0) - e(p_x^0, p_y^0, u^0) \\ &= \int_{p_x^0}^{p_x'} x^c(p_x, p_y^0, u^0) dp_x \end{aligned}$$

Refer to the graph on page 4 of the notes on Consumer Surplus (following page 83 of the notes) to see graphically why $CV > \Delta CS > EV$.

2.2 Question 5

To find the cost function, the firm will solve

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \text{ s.t. } y = \{\min[x_1/\alpha, x_2/\beta]\}^{1/2}$$

The minimum cost mix of inputs to acheive output level y will occur when $y^2 = x_1/\alpha = x_2/\beta$ thus

$$\begin{aligned} x_1^c &= \alpha y^2 \\ x_2^c &= \beta y^2 \\ c(y, w_1, w_2) &= y^2(w_1\alpha + w_2\beta) \end{aligned}$$

this makes marginal cost

$$c_y(y, w_1, w_2) = 2y(w_1\alpha + w_2\beta)$$

tho find the optimal level of output the firm will set marginal revenue (which equals price) equal to marginal cost

$$\begin{aligned} p &= 2y(w_1\alpha + w_2\beta) \\ y(p, w_1, w_2) &= \frac{p}{2(w_1\alpha + w_2\beta)} \end{aligned}$$

3 Part III

3.1 Question 6

3.1.1 (a)

Each lawyer will want to maximize his or her own utility taking the partners' hours as fixed. Thus, lawyer 1 will solve

$$\max_{h_1} y_1 - \alpha h_1$$

or

$$\max_{h_1} \frac{1}{2}(h_1 + h_2)^{1/2} - \alpha h_1$$

the FOC is

$$\begin{aligned} \frac{1}{4}(h_1 + h_2)^{-1/2} - \alpha &= 0 \\ h_1 &= \frac{1}{16\alpha^2} - h_2 \end{aligned}$$

thus by symmetry the reaction functions (or best response functions of the lawyers are

$$\begin{aligned} h_1 &= \psi_1(h_2) = \frac{1}{16\alpha^2} - h_2 \\ h_2 &= \psi_2(h_1) = \frac{1}{16\alpha^2} - h_1 \end{aligned}$$

The Cournot equilibrium is a Nash equilibrium. Using symmetry to solve

$$\begin{aligned} h_s^N &= \psi_s(h_s^N) = \frac{1}{16\alpha^2} - h_s^N \\ h_s^N &= \frac{1}{32\alpha^2} = h_1^C = h_2^C \end{aligned}$$

thus total hours are

$$h_{total}^C = h_1^C + h_2^C = \frac{1}{16\alpha^2}$$

each partener will get utiltiy level

$$\begin{aligned} U* &= \frac{1}{2}(h_{total}^C)^{1/2} - \alpha \frac{1}{2} h_{total}^C \\ &= \frac{1}{4\alpha} - \frac{1}{16\alpha} \\ &= \frac{3}{32\alpha} \end{aligned}$$

3.1.2 (b)

This is the monopoly solution. Lawyer 1 will now get to set the hours of both lawyers but has to make sure that she pays lawyer 2 enough so that he will achieve utility level $U* = \frac{3}{32\alpha}$. To do so, lawyer 1 will solve

$$\max_{h_1,h_2} \pi(h_1,h_2) - y_2 - \alpha h_1 \text{ s.t. } y_2 = \frac{3}{32\alpha} + \alpha h_2$$

substituting the constraint in

$$\max_{h_1,h_2} (h_1+h_2)^{1/2} - \frac{3}{32\alpha} - \alpha h_2 - \alpha h_1$$

Note that the hours enter as $h_1 + h_2$ everywhere in the maximization thus we can consider a single choice variable $h_{total} = h_1 + h_2$.

$$\max_{h_{total}} (h_{total})^{1/2} - \frac{3}{32\alpha} - \alpha h_{total}$$

the FOC is

$$\begin{aligned} \frac{1}{2}(h_{total})^{-1/2} - \alpha &= 0 \\ h_{total} &= \frac{1}{4\alpha^2} \end{aligned}$$

which is greater than under the Cournot solution. Note that total utility is

$$\begin{aligned} U_1 + U_2 &= Y_1 + y_2 - \alpha h_1 - \alpha h_2 \\ &= h_{total}^{1/2} - \alpha h_{total} \\ &= \frac{1}{2\alpha} - \frac{1}{4\alpha} \\ &= \frac{1}{4\alpha} \\ &= \frac{8}{32\alpha} \end{aligned}$$

which is larger than total utility under the Cournot solution

$$2U^* = \frac{6}{32\alpha}$$

The reason that total utility and hours are greater when one lawyer sets both of the lawyers' hours is because the coordination problem is overcome. Under the Cournot solution each lawyer is only taking their own cost of working less into account and weighing it against the benefit of having more leisure time. You can see this by considering whether the lawyers have an incentive to deviate from the monopoly level of hours if they each set their own hours. Consider lawyer 2

$$\frac{\partial U_2}{\partial h_2} = \frac{1}{4}(h_1^m + h_2^m)^{-1/2} - \alpha = \frac{2\alpha}{4} - \alpha = -\frac{\alpha}{2} < 0$$

implying that the gain in utility from additional wages due to a marginal reduction in hours outweighs the loss in utility from decreased income. Thus when each lawyer chooses their own hours they will choose to lower hours than the monopoly solution because they do not take the negative externality that their hours reduction imposes on the other partner (in terms of lost income) into account.

Note, the owner is completely indifferent between having lawyer 2 work any range between 0 and $\frac{1}{4\alpha^2}$ hours as long as the total hours of the two lawyers are $\frac{1}{4\alpha^2}$. To see this note that

$$\begin{aligned}
U_1 &= h_{total}^{1/2} - y_2 - \alpha h_1 \\
&= h_{total}^{1/2} - \left(\frac{3}{32\alpha} + \alpha h_2\right) - \alpha h_1 \\
&= h_{total}^{1/2} - \left(\frac{3}{32\alpha} + \alpha(h_{total} - h_1)\right) - \alpha h_1 \\
&= h_{total}^{1/2} - \frac{3}{32\alpha} - \alpha h_{total} + \alpha h_1 - \alpha h_1 \\
&= h_{total}^{1/2} - \frac{3}{32\alpha} - \alpha h_{total}
\end{aligned}$$

For example, if $h_0 = 0$ then $h_1 = \frac{1}{4\alpha^2}$ thus $y_2 = \frac{3}{32\alpha} + \alpha h_2 = \frac{3}{32\alpha}$. Then

$$\begin{aligned}
U_1 &= h_{total}^{1/2} - y_2 - \alpha h_1 \\
&= \frac{1}{2\alpha} - \frac{3}{32\alpha} - \alpha\frac{1}{4a^2} \\
&= \frac{16 - 3 - 8}{32\alpha} = \frac{5}{32\alpha}
\end{aligned}$$

On the other hand, if $h_1 = 0$ then $h_2 = \frac{1}{4\alpha^2}$ thus $y_2 = \frac{3}{32\alpha} + \alpha h_2 = \frac{3}{32\alpha} + \frac{1}{4\alpha} = \frac{11}{32\alpha}$. Then

$$\begin{aligned}
U_1 &= h_{total}^{1/2} - y_2 - \alpha h_1 \\
&= \frac{1}{2\alpha} - \frac{11}{32\alpha} \\
&= \frac{16 - 11}{32\alpha} = \frac{5}{32\alpha}
\end{aligned}$$