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Microeconomic Theory
Economics 101A
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**Economics 101A: Microeconomic Theory**
**Section Notes for Week 12**

# 1 Efficient Insurance

We will show that if actuarially fair insurance is offered, then it will be optimal for consumers to fully insure. Actuarially fair insurance is an insurance contract where the expected cost to the consumer and the expected profit of the firm selling the contract are both zero. The model set up is:

- The consumer begins with income of $y_0$.
- The consumer has probability, $p > 0$ of getting into an accident which will cause the consumer to incur a loss of income of $L$.
- With probability $1 - p$ there won't be an accident.
- An insurance contract is available which has a payout of $-\pi C$ if no accident occurs and a payout of $C - \pi C$ if an accident happens.
- We assume that the consumer is an expected utility maximizer and thus picks the level of coverage $C$ to solve

$$\max_C E[u(\tilde{y})] = \max_C \{pu(y_0 - L + C - \pi C) + (1-p)u(y_0 - \pi C)\}$$

  where $\tilde{y}$ is a random variable representing the consumer's income.
- We can think of the amount of income in the accident state, $s_1$ and the no accident state, $s_2$ as state contingent goods. If $s_1$ occurs then the consumer has income $x_1 = y_0 - L + C - \pi C$ and if $s_2$ occurs then the consumer has income $x_2 = y_0 - \pi C$.

To solve the consumer's problem we find the first order condition

$$\begin{aligned} \frac{\partial E[u(\tilde{y})]}{\partial C} &= p(1-\pi)u'(y_0 - L + C - \pi C) - (1-p)\pi u'(y_0 - \pi C) = 0 \\ \frac{u'(y_0 - \pi C)}{u'(y_0 - L + C - \pi C)} &= \frac{\frac{p}{1-p}}{\frac{\pi}{1-\pi}} \end{aligned}$$

This relationship provides the same intuition that we saw graphically in lecture yesterday.

1. If firm's offer actuarially fair insurance then $\pi = p$ and the odds ratio on the right hand side equals one. Since $u'(\cdot) > 0$ and $u''(\cdot) < 0$ everywhere marginal utility is always decreasing in income, thus $u'(x_2) = u'(x_1)$ implies that $x_1 = x_2$, which means that income is the same no matter which state of the world occurs. The consumer is fully insured.

2. If less than actuarially fair insurance is offered, then $\pi > p$ hence the right hand side will be less than one. This implies that $u'(x_2) < u'(x_1)$ thus $x_2 > x_1$ meaning that income is higher if no accident occurs or that the consumer is less than fully insured.

3. Similarly, if more than actuarially fair insurance is offered, then $\pi < p$ hence the right hand side will be greater than one. This implies that $u'(x_2) > u'(x_1)$ thus $x_2 < x_1$ meaning that income is higher if the accident occurs or that the consumer is more than fully insured.

## 2 Adverse Selection

The setup for adverse selection is the same as the previous model, except now there are two types of individuals with different probabilities of having an accident. We assume that the individuals have the exact same expected utility function which implies that they have the same degree of risk aversion. The low type has an accident with probability $p_L$ and the high type has an accident with probaility $p_H$ where $p_L < p_H$.

### 2.1 Efficient Solution: Perfect Monitoring

First we consider the case where insurance firms can tell which type of consumer they are facing (this assumption is sometimes referred to as perfect monitoring). In this case the insurance agency will set a premium for the low type $\pi_L = p_L$ and a premium for the high type $\pi_H = p_H$. Note that this implies that for the low type

$$\frac{u'(y_0 - \pi_L C)}{u'(y_0 - L + C - \pi_L C)} = \frac{\frac{p_L}{1-p_L}}{\frac{\pi_L}{1-\pi_L}} = 1$$

which means that $x_1 = x_2$ for the low type (they insure fully). Similarly for the high type

$$\frac{u'(y_0 - \pi_H C)}{u'(y_0 - L + C - \pi_H C)} = \frac{\frac{p_H}{1-p_H}}{\frac{\pi_H}{1-\pi_H}} = 1$$

which implies that the high type also fully insures. Note, however, that income in both states is lower for the high type than it is for the low type since $\pi_H C > \pi_L C$.

### 2.2 Failure of Efficiency

Next we consider the case where firms can't tell whether they are dealing with a consumer who has a low probability of getting into an accident or a high probability of getting into an accident. The model is the same except now we assume that $\lambda$ is the proportion of the population that is low risk and the $1 - \lambda$ is the proportion of the population that is high risk.

The efficient contract where the firm offers two premiums $\pi_L = p_L$ and $\pi_H = p_H$ will no longer work. To see this, note that all high risk type consumers will pretend to be low risk since in order to get the lower premium, thus the firm's profits will be

$$E[profits] = \lambda[p_L((\pi_L - 1)C + (1 - p_L)\pi_L C] + (1 - \lambda)[p_H((\pi_L - 1)C + (1 - p_H)\pi_L C]$$

since $\pi_L = p_L$ the first term equals zero. Thus,

$$\begin{aligned} E[profits] &= (1-\lambda)[p_H((\pi_L - 1)C + (1-p_H)\pi_L C \\ &= C(1-\lambda)[p_H\pi_L - p_H + \pi_L - p_H\pi_L] \\ &= C(1-\lambda)[-p_H + \pi_L] \\ &= C(1-\lambda)[p_L - p_H] < 0 \end{aligned}$$

meaning that the firms will choose to go out of business, hence the efficient contracts outcome that we found under perfect monitoring is not an equilibrium.

## 2.3 Other Pooling Equilibria?

Are there any coverage level and premium combinations $(\pi^P, C^P)$ that an insurance company could offer to both types and still make a profit in a competitive industry? No.

For the firm to be able to earn zero profits and stay in business, it must be able to subsidize its losses from the high risk types with gains from the low risk types. However, $p_H > p_L$ implies that the MRS of the high types is always greater than the MRS of the low types. Thus, for any possible pooling contract, a rival firm can offer another contract that is better for the low types but worse for the high types. This contract will pull low type business away from the first firm, thus causing them to earn negative profits and go out of business. This argument can be made for every possible pooling contract, thus there are no equliibirum pooling contracts.

Draw Graph.

## 2.4 Separating Equilibria

Even though pooling equilibria do not exist, there are separating equilibria where one premium / coverage combination is chosen by the low risk types $(\pi_L^S, C_L^S)$ and a different combination is chosen by the high risk types $(\pi_H^S, C_H^S)$. For a separating equilibrium to be stable, it must be the case that the high risk types have no incentive to select the contract intended for the low risk types, and the firm offereing the contracts must earn expected profits $\geq 0$ so it will stay in business.

Competition among firms for the high risk consumers will ensure that they are able to buy their efficient contract, thus $(\pi_H^S, C_H^S) = (p_H, L)$.

To ensure that the high risk types don't want to switch to the contract intended for the low risk types we must make sure that the expected utility of the high risk types is lower when they choose $(\pi_L^S, C_L^S)$ than it is when they choose $(p_H, L)$ the contract intended for them. Thus the boundary for a feasible separating contract for the low risk type is implicitly defined by the equation

$$\begin{aligned} & p_H u(y_0 - L + C_L^S - \pi_L^S C_L^S) + (1-p_H)u(y_0 - \pi_L^S C_L^S) \\ =\ & p_H u(y_0 - L + L - p_H L) + (1-p_H)u(y_0 - p_H L) \\ =\ & u(y_0 - p_H L) \end{aligned}$$

The other restriction comes from the fact that firms must be making at least zero expected profits. The firms expected profits are

$$\begin{aligned} E[profits] &= \lambda[p_L(\pi_L^S - 1)C_L^S + (1 - p_L)\pi_L^S C_L^S] + 0 \\ &= \lambda C_L^S[p_L\pi_L^S - p_L + \pi_L^S - p_L\pi_L^S] \\ &= \lambda C_L^S[\pi_L^S - p_L] \geq 0 \end{aligned}$$

Thus zero expected profits occurs when $\pi_L^S = p_L$ making insurance actuarially fair. Substituting back into out condition for the separating equilibrium to be feasible we get

$$p_H u(y_0 - L + C_L^S - p_L C_L^S) + (1 - p_H)u(y_0 - p_L C_L^S) = u(y_0 - p_H L)$$

an equation which implicitly defines $C_L^S$ in terms of known parameters.