Department of Economics
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Microeconomic Theory
Economics 101A
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Economics 101A: Microeconomic Theory
Section Notes for Week 7

1 Applied Competitive Analysis

The competitive market model is a useful framework to analyze the impact of a number of governmental policy actions.

1.1 Competitive Equilibrium

Asumptions:

- $D(p)$, the aggregate market demand curve is downward sloping ($\frac{dD(p)}{dp} < 0$).
- $S(p)$, the aggregate market supply curve is upward sloping ($\frac{dS(p)}{dp} > 0$).
- The supply and demand curves describe the market for a specific homogenous good.
- Markets are perfectly competitive so that all consumers face the same demand price, p_D and all suppliers face the same supply price, p_S.

In general, we can find the quanities that are demanded and supplied by reading them from demand and supply curves at the appropriate prices

$$\begin{aligned} Q_D &= D(p_D) \\ Q_S &= D(p_S) \end{aligned}$$

If there are no government interventions then the equilibrium price, p^* will be such that supply and demand are set equal

$$S(p^*) = D(p^*)$$

in this case we have

$$\begin{aligned} p_D &= p_S \\ Q_D &= Q_S \end{aligned}$$

For the rest of the analysis, let p_0, Q_0 represent the equilibrium price and quantity absent any governmental intervention so that before any intervention $p_0 = p_S = p_D$ and $Q_0 = Q_S = Q_D$.

1.2 Per Unit Tax

We can analyze the impact of a per unit tax, τ by noting that now demanders will have to pay the price that goes to the suppliers plus the tax that goes to the government so

$$p_D = p_S + \tau$$

(Note, a subsidy can be analyzed by setting $\tau < 0$). The equilibrium condition under a per unit tax can be written as

$$D(p^* + \tau) = S(p^*)$$

we can use comparitive statics to find the impact of a small tax, $d\tau$, on output relative to the equilibrium without a tax. The analysis proceeds by considering the tax, τ, as an exogenous parameter upon which the equilibrium price depends, $p^*(\tau)$. Thus, we can write the equilibrium condition as

$$D(p_S^*(\tau) + \tau) = S(p_S^*(\tau))$$

Taking the total derivative with respect to τ we get

$$\frac{dD}{dp_S^*}\left(\frac{dp_S^*}{d\tau} + 1\right) = \frac{dS}{dp_S^*}\frac{dp_S^*}{d\tau}$$

re-arranging gives

$$\frac{dp_S^*}{d\tau} = \frac{\frac{dD}{dp_S^*}}{\frac{dS}{dp_S^*} - \frac{dD}{dp_S^*}}$$

Equivalently, we could express the equilibrium condition as

$$D(p_D^*(\tau)) = S(p_D^*(\tau) - \tau)$$

taking its total derivative and re-arranging leads to

$$\frac{dp_D^*}{d\tau} = \frac{\frac{dS}{dp_D^*}}{\frac{dS}{dp_D^*} - \frac{dD}{dp_D^*}}$$

thus the impact of the tax on equilibrium quantities is

$$\begin{aligned} \frac{dQ_S^*(p_S^*(\tau))}{d\tau} &= \frac{dS}{dp_S^*}\frac{dp_S^*}{d\tau} \\ &= \frac{\frac{dS}{dp_S^*}\frac{dD}{dp_S^*}}{\frac{dS}{dp_S^*} - \frac{dD}{dp_S^*}} \end{aligned}$$

Note that since $p_S^* + \tau = p_D^*$

$$\begin{aligned} \frac{dS}{dp_S^*} &= \frac{dS}{dp_D^*} \\ \frac{dD}{dp_D^*} &= \frac{dD}{dp_S^*} \end{aligned}$$

thus we can write

$$\frac{dQ_S^*}{d\tau} = \frac{\frac{dS}{dp}\frac{dD}{dp}}{\frac{dS}{dp} - \frac{dD}{dp}} = \frac{dQ_D^*}{d\tau}$$

1.2.1 Elasticities

To express the above equation in terms of elasticities we can multiply the numerator and denominator by $\frac{p_0}{Q_0}$ to get

$$\frac{dp_S^*}{d\tau} = \frac{\eta}{\sigma - \eta}$$

and then multiply both sides by $\frac{d\tau}{p_0}$

$$\frac{dp_S^*}{p_0} = \frac{\eta}{\sigma - \eta}\frac{d\tau}{p_0}$$

similarly

$$\frac{dp_D^*}{p_0} = \frac{\sigma}{\sigma - \eta}\frac{d\tau}{p_0}$$

and

$$\frac{dQ_S^*}{d\tau} = \frac{\frac{dS}{dp}\eta}{\sigma - \eta}$$

and then multiply both sides by $\frac{d\tau}{Q_0}$

$$\begin{aligned}
\frac{dQ_S^*}{Q_0} = \frac{dQ_D^*}{Q_0} = \frac{dQ^*}{Q_0} &= \frac{\frac{dS}{dp}\eta}{\sigma - \eta}\frac{d\tau}{Q_0} \\
&= \frac{\frac{p_0}{Q_0}\frac{dS}{dp}\eta}{\sigma - \eta}\frac{d\tau}{p_0} \\
&= \frac{\sigma\eta}{\sigma - \eta}\frac{d\tau}{p_0}
\end{aligned}$$

1.2.2 Dead Weight Loss

The dead weight loss (DWL) of a tax is given by the area of the region that lies between the demand curve, supply curve, and vertical line at $Q = Q_0$. DWL can be approximated by using the formula for the area of a triangle (using $-dQ$ as the length and $d\tau$ as the height).

$$\begin{aligned}
DWL &\approx -\frac{1}{2}d\tau dQ \\
&= -\frac{1}{2}d\tau\frac{\sigma\eta}{\sigma - \eta}\frac{Q_0 d\tau}{p_0} \\
&= -\frac{1}{2}(d\tau)^2\frac{\sigma\eta}{\sigma - \eta}\frac{Q_0}{p_0}
\end{aligned}$$

or DWL express as a fraction of revenue, $R = pQ$

$$\frac{DWL}{R} \approx -\frac{1}{2}\left(\frac{d\tau}{p_0}\right)^2 \frac{\sigma\eta}{\sigma - \eta}$$

1.3 Government Purchases

We can also analyze the impact of a government purchase of the good, G. The equilibrium conditions are now

$$p_D = p_S = p^*$$

$$D(p^*) + G = S(p^*)$$

Again using comparative statics we find that

$$\begin{aligned} \frac{dD}{dp^*}\frac{dp^*}{dG} + 1 &= \frac{dS}{dp^*}\frac{dp^*}{dG} \\ \frac{dp^*}{dG} &= \frac{1}{\frac{dS}{dp} - \frac{dD}{dp}} \end{aligned}$$

$$\begin{aligned} \frac{dQ_S^*}{dG} &= \frac{dQ_S^*}{dp^*}\frac{dp^*}{dG} = \frac{\frac{dS}{dp}}{\frac{dS}{dp} - \frac{dD}{dp}} \\ \frac{dQ_D^*}{dG} &= \frac{dQ_D^*}{dp^*}\frac{dp^*}{dG} = \frac{\frac{dD}{dp}}{\frac{dS}{dp} - \frac{dD}{dp}} \end{aligned}$$

1.3.1 Elasticities

$$\frac{dp^*}{p_0} = \frac{1}{\sigma - \eta}\frac{dG}{Q_0}$$

$$\begin{aligned} \frac{dQ_S^*}{Q_0} &= \frac{\sigma}{\sigma - \eta}\frac{dG}{Q_0} \\ \frac{dQ_D^*}{Q_0} &= \frac{\eta}{\sigma - \eta}\frac{dG}{Q_0} \end{aligned}$$

1.3.2 Dead Weight Loss

The dead weight loss (DWL) of a government purchase is given by the area of the region that lies above the horizontal line through the old equilibrium price, p_0, below the demand curve, and between the vertical lines at Q_D^* and Q_0 and below the supply curve between the vertical lines at Q_0 and Q_S^*.

We can approximate the sum of these to triangles by half their height times the sum of their lengths

$$\begin{aligned} DWL &\approx \frac{1}{2} dp^*(dQ_S^* - dQ_D^*) \\ &= \frac{1}{2} \frac{1}{\sigma - \eta} \frac{dG}{Q_0} p_0 \left(\frac{\sigma}{\sigma - \eta} dG - \frac{\eta}{\sigma - \eta} dG \right) \\ &= \frac{1}{2} \left(\frac{1}{\sigma - \eta} \right) \frac{p_0}{Q_0} (dG)^2 \end{aligned}$$

we can also express dead weight loss as a fraction of revenue, $R = pQ$

$$\frac{DWL}{R} \approx \frac{1}{2} \left(\frac{1}{\sigma - \eta} \right) \left(\frac{dG}{Q_0} \right)^2$$