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Microeconomic Theory
Economics 101A
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Economics 101A: Microeconomic Theory
Section Notes for Week 9

1 Spence's (1973) Job Market Signalling Game

This example is taken from the book "Game Theory for Applied Economists" by Robert Gibbons. It is an example of a dynamic game with incomplete information. Although I will not formally define the equilibrium, the sketch I provide is based upon an equilibrium concept called Perfect Bayesian Equilibrium.

1.1 Setup

There are two possible types of workers. High productivity workers denoted H and low productivity workers denoted L.

1. Nature determines whether the productive ability of the worker, η. $\eta = H$ with probability q and $\eta = L$ with probability $1 - q$ (note: $0 < q < 1$).
2. The worker learns his or her type and chooses how many years of schooling to complete $S \geq 0$.
3. The "labor market" observes the worker's level of schooling but not the worker's type and makes a wage offer equal to his or her expected productivity.
4. The worker accepts the offer. (Note the worker will always accept the offer since $w - c(\eta, S) > 0 - c(\eta, S)$.

The payoff to the worker will be $w - c(\eta, S)$. The term $c(\eta, S)$ represents the cost (both mental and monetary) that a worker of type η must incur to complete S years of schooling.

The payoff to the firm will be $y(\eta, S) - w$ if the firm hires the worker or 0 if the firm does not hire the worker.

1.2 Assumptions For a Signalling Equilibrium

The critical assumption is that the marginal cost of schooling is greater for the low type of worker than for the high type. This condition can be expressed as

$$\frac{\partial c(L, S)}{\partial S} > \frac{\partial c(H, S)}{\partial S} \forall S$$

Graphically, the workers indifference curves can be represented in (w, S) space. The above condition means that for particular level of schooling a type L worker will always have a steeper indifference curve than a type H worker. The implication is that type L workers will need a larger rise in wages to compensate them for an increase schooling level from S^0 to S' than is required by type H workers.

Draw Graph.

A second assumption is that competition among firms drive their expected profits to zero, which is why the worker is offered a wage equal to his or her expected productivity. Let $\mu(\eta|S)$ denote labor markets's beliefs about whether the the worker is type η conditional on observing that he or she has gone to school for S years. Specifically, the market believes that a worker with S years of schooling is type H with probability $\mu(H|S)$. Under this assumption the expected wage conditional upon years of schooling can be written as

$$w(S) = \mu(H|S)y(H,S) - (1 - \mu(H|S))y(L,S)$$

1.3 Complete Information Case

To analyze the equilibria, let's first think about how the game would play out under complete information, that is to say, if the market knew the type of worker it was offering a wage to. In this case

$$w(S) = y(\eta, S)$$

facing this wage, the worker would choose S to solve

$$\max_S y(\eta, S) - c(\eta, S)$$

let $S^*(\eta)$ denote the optimal level of schooling for a worker of type η. Then $w^*(\eta) = y(\eta, S^*(\eta))$. This solution can be represented graphically.

Draw graph.

1.4 Incomplete Information Case

Now we will revert back to the initial setup with incomplete information. In this case, the market knows that the initial probability of a type H worker is q. Then based upon the level of schooling that the worker chooses the market will update its assessment of the probability that the worker is type H. It is assumed that the market uses Bayes' Rules to update its belief $\mu(H|S)$ about the probability that the worker is type H.

There are several possible types of equilibria in this model: pooling, separating (or signalling), and hybrid. We will focus on the separating equilibrium, but first mention the pooling equilibrium. In the pooling equilibrium, the signal (years of schooling S) does not reveal any new information to the market, thus the markets belief remains $\mu(H|S) = q$.

In the separating equilibrium the signal is fully revealing, meaning that after observing the level of schooling that at worker has chosen, the market will know the worker's type with certainty. Two cases are possible. The first case called the "no-envy" case occurs when it is too expensive for the type L workers to aquire schooling level $S^*(H)$ even though doing so would allow them to masquerade as a type H worker and earn $w^*(H)$. This occurs when,

$$w^*(L) - c(L, S^*(L)) > w^*(H) - c(L, S^*(H))$$

Draw graph.

The second case is more interesting. In this case, the type H worker must invest in extra schooling (relative to both the perfect information case and the no-envy case) in order to dissuade the type L worker from masquerading as a type H worker. We will denote this new equilibrium choice of education for the type H worker as $S_S > S^*(H)$.

Draw graph.

One specification for the market's belief that supports this equilibrium is that the worker is type H if $S \geq S_S$ and type L if $S < S_S$. The market's beliefs are thus

$$\mu(H|S) = \begin{cases} 0 & \text{if } S < S_S \\ 1 & \text{if } S \geq S_S \end{cases}$$

the market's strategy will thus be to offer wage

$$w(S) = \begin{cases} y(L, S) & \text{if } S < S_S \\ y(H, S) & \text{if } S \geq S_S \end{cases}$$

Finally, we can briefly mention the hybrid equilibria. In these equilibria, one type chooses one level of schooling with certainty while the other randomizes between pooling with the first type and separating from the first type.