Dissertation Abstract: The Foundations of Conditional Probability

The goal of my dissertation is to understand in both formal and informal terms the relation between the concepts of conditional and unconditional probability, in the widely used interpretation of probability as degree of belief. (I argue that the details of the relation will differ in other interpretations of probability, but that many of my arguments should go through for objective epistemic interpretations as well as the subjective one I focus on.) Mathematically, conditional probability has been defined as a ratio, $P(A|B) = P(A\&B)/P(B)$. However, since there is a pre-theoretic notion of conditional degree of belief (approximately, the agent's degree of belief in A, on the *supposition* that B is true), there is a question as to whether this mathematical definition is adequate as an analysis. These arguments turn on the role of conditional probability plays in revising one's degrees of belief in light of new evidence, and also in clarifying the intuitive notions of support or confirmation.

Consider the case of a scientist hypothesizing about the value of a physical constant - there are infinitely many possible values, so unless there is some bias in her degrees of belief, any precise hypotheses H will have $P(H) = 0$. But then the degree of support her evidence gives will depend on $P(E\&H)/P(H)$, which involves dividing by 0, which is senseless. Many philosophers, following Popper, have used this sort of example to argue that analyzing conditional probability in terms of unconditional can't work, and that we should instead see *all* degrees of belief as conditional.

However, I argue that when all relevant epistemic norms are taken into account, this picture fares little better than the traditional mathematical account. In the end, I argue that conditional probability is not just a two place notion, involving propositions A and B, but also requires the specification of a set of "relevant alternatives" for B. That is, in order for an agent to revise her degrees of belief, it matters not just what she learned, but how she learned it. Thus, I claim that conditional probability cannot be taken as fundamental.

My main argument relies on a principle known as "conglomerability", which basically says that no potential outcome of an experiment can confirm a hypothesis unless some other potential outcome would disconfirm it. I then give examples of probability spaces in which there is no way to assign values $P(A|B)$ that conform to this principle, unless we allow a different value when the same proposition B is considered as an outcome of different experiments. I consider some alleged counterexamples to conglomerability, but argue that these cases are purely mathematical and can't actually represent an agent's degrees of belief.

In the early chapters I discuss the informal notions of conditional and unconditional degree of belief, to make sure that I have them properly set up to develop a mathematical account later on. Then I consider what mathematical principles are needed to properly respect these informal notions, and follow up with my arguments described above. Finally, I consider some alternative accounts and show how they fail.

Further Research Interests

I have three major areas of research interest, which have a lot of overlap. The central one is a continuation of my dissertation research, to understand more about the connections between different interpretations of probability, and the extent to which relations between conditional and unconditional probability in subjective probability can be extended to chance, logical, or frequency notions of probability.

Closely related is a continuation of work that I have been doing on the foundations of decision theory. I have been working on a paper arguing for a broader notion of rational decisions, which can apply in situations where expected utilities don't exist. My proposal starts with the basic notion of dominance, and then extends this decision procedure to more situations (including all situations handled by expected utility) by considering relations of indifference. I would like to develop these ideas further in light of what I say in "Strong and Weak Expectations", which suggests that gambles may be valued based on their hypothetical limiting behavior under repetition, rather than the axiomatic constraints that are used to prove representation theorems.

My other major series of interests is centered on the philosophy of mathematics. In "The Role of Axioms in Mathematics", I have argued that axioms allow mathematical research to progress despite a lack of consensus on foundational philosophical issues. I would like to see how this idea can be further extended, in particular arguing that very divergent views about mathematics (like ones that take mathematical objects to be real, and ones that take them to be merely "useful fictions") can both recommend inference to the best explanation as a procedure both for the conjecture-and-theorem aspect of mathematics and for the justification of axioms. This will involve both investigation into the nature of explanation in mathematics, and a connection to my work on probability in seeing how the notion of partial belief is appropriately extended to subject matters traditionally considered to be *a priori*. This latter issue is also involved in a paper I have been working on discussing "probabilistic proofs" and the role they do and should play in mathematics.