Purdue Problem of the Week

Gagan Tara Nanda
Senior at University of California, Berkeley
1927 Dwight Way, #102, Berkeley, CA 94704-1945

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1. Let the given number be $abcd$, in base 10 notation. We are given that $cd < 10$. The only possible choices for cd are 01, 03, 07, and 09, since a number ending in 00, 02, 04, 06, and 08 is even, and one ending in 05 is divisble by 5. Further, note that $a > 0$ since $N - n = 999$. We have $dcba - abcd = 999$. We have two possibilities:

$$d + 9 = a \qquad (1)$$
$$d + 9 = 10 + a \qquad (2)$$

Observe that if (1) is true, then $d = 0$, which leads to a contradiction. So (2) holds and $d = a + 1$. This implies either of the following:

$$(b - 1) - c = 9 \qquad (3)$$
$$(10 + b - 1) - c = 9 \qquad (4)$$

Again, (3) cannot hold because that implies $b - c = 10$, but $b - c \leq 9 - 0 = 9$. So (4) holds and we get $b = c$. From our choices for cd, we conclude that $b = c = 0$. Our required number thus has the form $a00\,(a + 1)$. If $cd = 07$, then $abcd = 6007 > 6000$, and if $cd = 09$, then $abcd = 8009 > 6000$. Hence we deduce that $cd = 03$, and the required prime is 2003.