Omitted Variable & Diff-in-Diff

Difference of two differences.
Assume treatment is taking a drug or going to school.
$T_i = 1$ if take magic drug
X_i =hours of exercise you do
Y_i = health (0-100)

$$Y_i = a + bT_i + cX_i + e_i$$

So now we can take conditional expections

$$\mathbb{E}[Y_i|T_i = 1] - \mathbb{E}[Y_i|T_i = 0] = ATE$$

Let's do it:
$\mathbb{E}[Y_i|T_i = 1] = a + b + c\mathbb{E}[X_i|T_i = 1] + \mathbb{E}[e_i|T_i = 1]$
$\mathbb{E}[Y_i|T_i = 0] = a + 0 + c\mathbb{E}[X_i|T_i = 0] + \mathbb{E}[e_i|T_i = 1]$
Let's assume $\mathbb{E}[e_i|T_i = 1] = \mathbb{E}[e_i|T_i = 1] = 0$
We are left with

$$b + c\left(\mathbb{E}[X_i|T_i = 1] - \mathbb{E}[X_i|T_i = 0]\right)$$

What's with this last term. If people who exercise (and care more about their health) also are more likely to take the pill, then

$$\mathbb{E}[X_i|T_i = 1] - \mathbb{E}[X_i|T_i = 0] > 0$$

Our goal is get rid of this term. Let's assume there are two time periods. 0=1980 1=1985

$$\mathbb{E}[Y_{it}|T_{i1}]$$

$$\mathbb{E}[Y_{i0}|T_i = 1] - \mathbb{E}[Y_{i0}|T_i = 0]$$

$\mathbb{E}[Y_{i0}|T_i = 1] = a + b(0) + c\mathbb{E}[X_{i0}|T_{i1} = 1]$
$\mathbb{E}[Y_{i0}|T_i = 0] = a + 0 + c\mathbb{E}[X_{i0}|T_{i1} = 0]$
Subtract and get
$c\left(\mathbb{E}[X_{i0}|T_{i1} = 1] - \mathbb{E}[X_{i0}|T_{i1} = 0]\right)$
Now the big diff-in-diff

$$\begin{aligned}
\mathbb{E}[Y_i|T_i &= 1] - \mathbb{E}[Y_i|T_i = 0] - \left(\mathbb{E}[Y_{i0}|T_i = 1] - \mathbb{E}[Y_{i0}|T_i = 0]\right) \\
&= b + c\left(\mathbb{E}[X_i|T_i = 1] - \mathbb{E}[X_i|T_i = 0]\right) - c\left(\mathbb{E}[X_{i0}|T_{i1} = 1] - \mathbb{E}[X_{i0}|T_{i1} = 0]\right) \\
&= b
\end{aligned}$$