Go Figure
It's all a Math Problem
Edition 2 Block G May 2011

It Takes Two to Tango

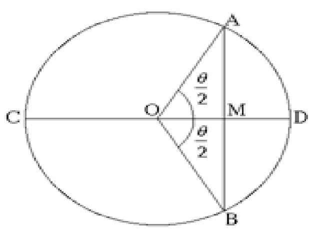
Emily Tsai and Nicole Ellis

It only takes two pieces of information (one angle and one side) to find a distance.

History:
Trigonometry comes from a Greek word "trigonometria" put together from these 3 words: Tri (three), gonia (angle), and metro (measure).
Hipparchus is the founder of trigonometry. He was the first Greek mathematician to study triangular geometry. Other people have additional research to Hipparchus' work, but the two people that contributed the most are were Menelaus and Ptolemy. Menelaus invented a couple of triangle properties, and contributed a big part in spherical trigonometry. Ptolemy was a Greek astronomer and he was the first mathematician to complete the tables of chords, which allowed early astronomers to calculate the distance between two stars based on the observed angle between them.

The Muslims, Chinese, Indians, and Babylonians each had their own information that made their way into trigonometry. The Muslims were the ones that introduced the tangent function, and the Indians made the cosine tables.

Trigonometry is "the branch of mathematics that deals with the relationships between the sides and the angles of triangles and the calculations based on them, particularly the trigonometric functions." It only takes knowing one angle and one side inside a right triangle to completely solve for all sides and all angles. *Continued on Page 4*



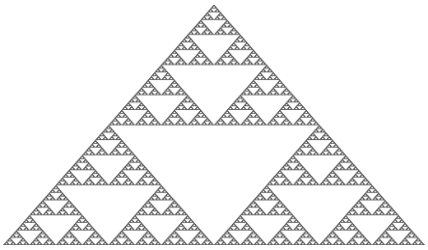
This diagram illustrates a famous trigonometric table created by Ptolemy, inspired by Hipparchus. The "table of chords" allows you to look up how long the chord AB will be, given the angle AOB and the size of the radius.
(Picture from http://hypertextbook.com/eworld/chords.shtml)

Patterns of Life By María-José Alvarez and Sofía Espinoza

Have you ever noticed the intricate designs around you? You would be amazed to know how many things are made out of tessellations. Characteristics of tessellations include geometric figures without any gaps or overlaps in a repetitive pattern. The repetitive patterns are made by geometric translations such as rotation (rotating the figure), reflection (reflecting it along an axis), or dilation (simple scaling using a multiplier or "scale factor.") *Continued on Page 5*

The Magic of Fractals

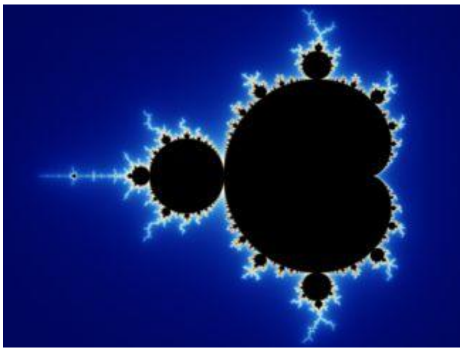
Daniella Salaverria and Fernanda Alfaro



This simple fractal shows how the triangle repeats itself in the property called self-similarity. This fractal is named the Sierpinski Triangle. Notice that the tiny triangles still resemble the whole.

Picture from http://eldar.mathstat.uoguelph.ca/dashlock/ftax/GSF.html.

According to Benoit Mandelbrot, a fractal is "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole." This description has a property called "Self-Similarity": a fractal shows us the same repeating shape as you zoom in on the original shape.



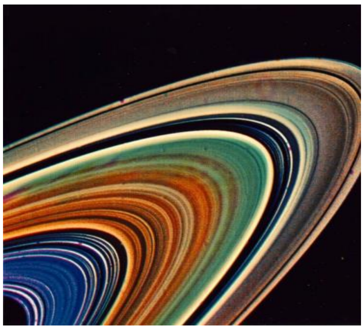
This is the "Mandelbrot Fractal" it is one of the most recognizable fractals.

Picture from http://www.logicnest.com/archives/tag/fractal.

Benoit Mandelbrot started working upon Lewis Fry Richardson's work on "Self Similarity" and "Fractional Dimension" in the 1960s. Richardson's work was what inspired Mandelbrot to work on fractals. By observing images, Mandelbrot realized that there existed a repeating pattern within the image. Later in 1975 he named a new term to refer to the images with repeating patterns that resembled the whole: Fractals. Within just 30 years, scientists have found numerous applications of fractals.

Fractals in Astronomy

Fractals are used in astronomy. Fractals were used to determine the number of rings in Saturn. At first, early astronomers thought that Saturn had only one ring, but they later discovered it in fact had two concentric rings. The Voyager spacecraft was the one that proved that Saturn had billions of small rings. Astronomers then understood that Saturn's rings were an example of the Cantor Set fractal and are therefore infinite. The Cantor Set is a fractal that keeps breaking down lines into pieces, as you can see in the picture below. The Cantor set is infinite, since the line always is broken down into pieces and that pattern is never ending. If you zoom in on the fractal, there will always be many smaller line segments. Saturn's rings are a Cantor set, because each ring becomes several thinner rings as you zoom in. Therefore, there is an infinite number of rings around Saturn.

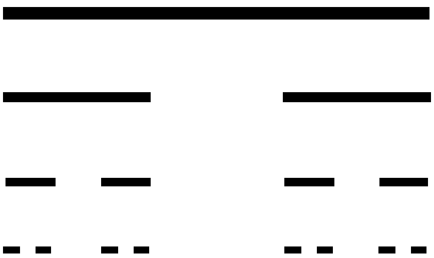


Picture from http://solarsystem.nasa.gov/planets/profile.cfm?Object=

The cantor set is the explanation of why Saturn's rings are an example of fractals, and therefore they are infinite.

Fractals in Earth Science

Fractals can also be used in earth science to predict natural disasters. Scientists can use past fractal measurements to determine future natural disaster like hurricanes, fires, volcanic eruptions, and weather. For example, to predict rainfall, scientist can map rainfall in the computer. When mapping rainfall, the mapped data repeats itself in a pattern that creates a fractal. In the future, this pattern can be used for predicting weather, since future natural events will follow similar patterns. Christopher Barton, who has been using this new technology, describes the usefulness of fractals: "They (scientists) can measure past events like a hurricane and then apply fractal mathematics to predict future hurricane events." With this new technology you can help many people by giving them knowledge of the future catastrophes that can occur.



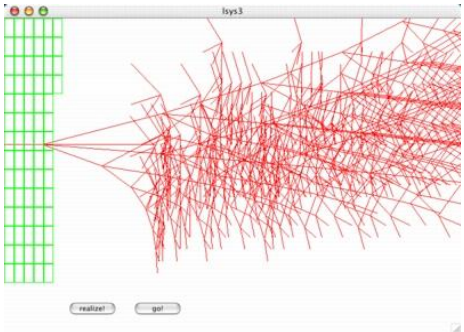
Cantor

Picture from http://lavacaesferica.com/2011/04/sobre-los-fractales-y-su-relacion-con-la-naturaleza-i/.

Fractals in Music

Some musicians have used fractals to create music. Today two main forms are used in musical applications -- the L-System and the Fractal Motion. In the L-System, you start with a "short string of symbols" and repeat it in different forms, making it an example of a fractal. In fractal motion you modify a line in a repeated way and that coverts it into a fractal. These two ways create different types of music.

Picture from http://music.columbia.edu/cmc/courses/g6610/fall2003/week5/index.html.



In this picture we visualize the L System music with computers. You can observe the repeating pattern.

Website information
http://eldar.mathstat.uoguelph.ca/dashlock/ftax/GSF.html

http://www.logicnest.com/archives/tag/fractal

http://solarsystem.nasa.gov/planets/profile.cfm?Object=Saturn&Display=Rings

http://lavacaesferica.com/2011/04/sobre-los-fractales-y-su-relacion-con-la-naturaleza-i/

http://www.tursiops.cc/fm/#howis

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http://www.sciencedaily.com/releases/2002/01/020131073853.htm

http://www.prairiehill.org/technology/student_work/fractals1_0708/bakmnkfractals.html

Continuations From the Front Page

It takes two to Tango.

Trigonometry and Physics

Trigonometry is often applied in physics. An object is in equilibrium and therefore not being forced to change its motion when all the external forces acting upon it add up to zero. When you know how much the angle of the ramp measures and you know how much the object in the inclined plane weighs; you are able to find the two forces acting upon the object.

Physicists use trigonometry to calculate the forces that will cause an object's change in position.

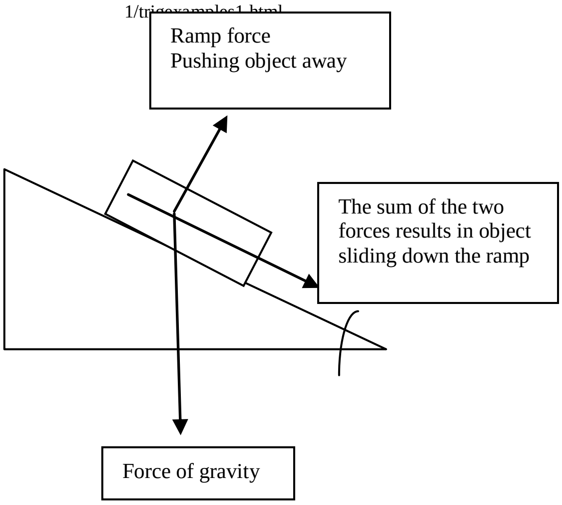
Other Applications

Trigonometry is very important in our world. Its applications extend far beyond what we study in geometry. It can be used in many ways and for many reasons. The study of trigonometry is the basic knowledge for many important jobs. Its importance is reflected in our everyday life. It would not be possible to construct cars or build houses without all types of engineering, which rely on the physics concepts that apply trigonometry to analyze physical forces.

Even though trigonometry involves the use of triangles, its applications extend beyond geometry. Fashion designers use trigonometry to work out their designs for dresses and calculate how much fabric they would have to use. Sea voyagers and flight navigators use trigonometry to determine their destination. Trigonometry techniques are also used in satellite systems, naval and aviation industries, oceanography, land surveying, and in cartography. All of these applications primarily use trigonometry to determine the distance between two objects. Trigonometry is applied in engineering to design buildings, cars, ships, and planes. It is used in higher-level physics to calculate the properties of electric and magnetic fields. It is used in navigation and in predicting projectile motion. Even the design of musical chords and instruments, as well as lenses and optics, relies on the application of trigonometric concepts.

Website information:
1. http://www.amtsgym-sdbg.dk/as/venus/ven-dist.htm
2. Physics: principles and problems (book)
3. http://www.bcc.cuny.edu/MathematicsComputerScience/ate/gps.html
4. http://centraledesmaths.uregina.ca/RR/database/RR.09.01/trigexamples1.html
5. http://ualr.edu/lasmoller/trig.html
6. http://www.concordacademy.org/academics/PrimerTrigPhysics.pdf
7. http://www.mathworksheetscenter.com/mathtips/trigonometry.html

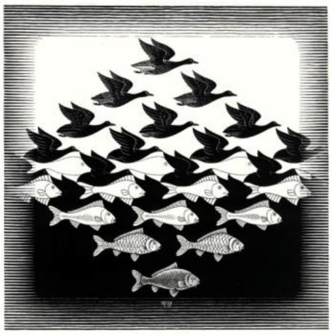
http://www.clarku.edu/~djoyce/trig/apps.html



The two forces acting upon the object are the gravitational force that pulls downward and the natural force that pushes up pulls perpendicular from the ground. In order to figure out how quickly the object will slide down the ramp, you would have to use trigonometry to "break down" gravity into smaller forces that point in different directions. The "net" force, or the sum of the forces, points diagonally down from the ramp as you can see in the picture.

Patterns of Life

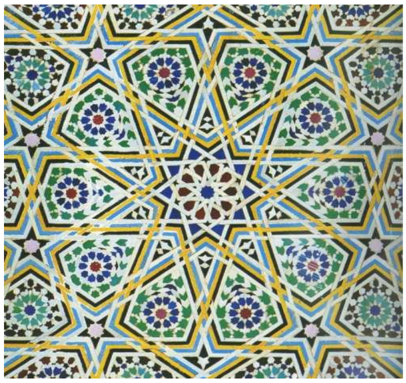
The word tessellation comes from the Latin word "tessera", which means "small stone cube"; cultural groups such as the Romans and Muslims to decorate their buildings, temples, i.e. an Islamic, and mosques using these small pieces of stone. In addition, the Moors people of Alhambra, Spain, used similar mosaics and tessellation techniques to cover their walls. Although mosaics can be tessellations, not all of them are



Sky and Water, by MC Escher.
http://burdr.tumblr.com/post/2721623086/sky-and-water-i-1938-woodcut-by-m-c-escher

because some contain irregularities or gaps.

Tessellations became very famous for the visual attention they attracted and soon artists, mathematicians, and architects began to imitate and create them. As a matter of fact, tessellations became popular way back in 3000 B.C. (when the Moors lived), but in the past these creations were more common in mosaics. Many centuries later, in the 18th and 19th centuries, people began experimenting with different figures and incorporated tessellations into modern art. For example, M.C. Escher, a renowned



http://www.thelck.com/patterns/images/rabat.jpg

mathematician and architect that lived during the mid 20th century, made a great contribution by introducing animals and irregular shapes to the idea of tessellations. Some of his most famous work includes "Liberation," "Cycles," and "Sky and Water," where he tessellated birds, fish, and other irregular shapes by metamorphosing them within the drawing. In the piece "Sky and Water," he starts with a flock of birds that turns into a school of fish by making the space between the characters tighter or looser. His work became famous because of the optical illusions he created and the depth and shades his creations contained, it appealed to people mathematically as well as artistically. Islamic art also frequently incorporates tessellation concepts; studies have shown that Islamic artisans in 1200 A.D. used a group of five different equilateral geometric figures to create complex tessellations. Muslims utilized *girih* tiles (decagon, pentagon, hexagon, bow-tie, and rhombus.) One of the most famous buildings with Islamic art is Gonbad-e Kabud, a tower in Maragha in the Azerbaijan province of Iran, shown in the picture to the right. The tiles fall effortlessly together without any gaps and repeat themselves; the smaller tiles are rotated and shifted to create a more irregular concept.

The work of the Moors, Romans, Muslims and M.C Escher has now become part of our daily lives, meaning that people have started noticing tessellations in nature. It is quite curious to notice the tessellations in pineapples and honeycombs, which occur naturally. Honeycombs and pineapples are examples of three-dimensional tessellations, but these can also be made through origami or are found in art. In reality, many of the 3D tessellations are an illusion, since rotating smaller 2D tessellations and then shading the outcome according to the lighting can make them. On the other hand, others are made by origami (the art of folding paper into different shapes). Making a 3D tessellation is not as hard as it seems, in truth, it basically uses two steps: folding and pushing upward to obtain a creative array of tangible three-dimensional shapes. It is also recommended to drift away from the guidelines and experiment with new tessellations. Even the simplest things in nature and other patterns around us are composed of tessellations, so next time you look around, don't forget to look for these! So, have you ever noticed the intricate designs around you, now?

Liberation, by MC Escher.

http://www.idproject.us/escherliberation.GIF

Gonbad-e Kabud, tower built in the Azerbaijan province of Iran.
Shown to the right of paragraph two.
Picture from Wikipedia

Sources:

http://www.tessella.com/about-us/what-are-tessellations/

http://sites.google.com/site/geometricarts/3dtessellation

http://cedison.wordpress.com/2009/02/22/3d-origami-tessellation-suggestions-and-other-stuff/

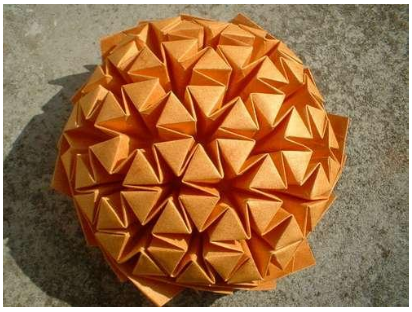
http://www.thelck.com/patterns/tenPointStar.html

http://tessellations.org/

http://www.princetonol.com/groups/iad/lessons/middle/tessell.htm

http://www.mcescher.com/

Honeycomb, naturally occurring tessellation.

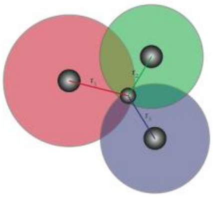


3D origami tessellation, triangular sphere.
http://static.zooomr.com/images/7404973_7e14a8cf00_o.jpg

Triangulation vs. Trilateration

What's the difference?

Alexander Ha and Carlos López



Triangulation was one of the first techniques ever invented for navigation. Sailors that navigated throughout the seas without radars or other devices used this method to calculate their location at sea. This technique is so efficient that it is still in use today. Triangulation is a technique used for calculating the location of a point by measuring angles to it from known points at either end of a base line. It applies the laws of sines by using two observers and the angles between them to find out the missing side. For example, astronomers use this technique to find out the distance of earth from a star. They simply gather the angle at which at star is seen on earth in different part of the year in which the earth is in the opposite side of its orbit. With that information and the distance of earth orbits around the sun, astronomers are able to get a pretty close estimate on what the distance the star has from the sun. When that information is revealed, all that is needed to find the earth's distance from the star is a little help from the Pythagorean Theorem.

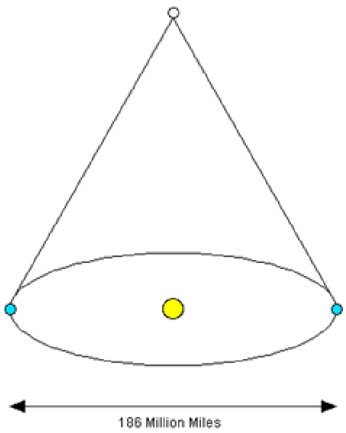
Trilateration is the process of locating a point through three or more vertices of a triangle by knowing each of their distance away from the point in question. Trilateration is not only used for navigation, but it is also used in GPS devices as well. The GPS receives signals from 3 different satellites that tell the GPS device how far away it is from all three or more satellites. When your GPS receives the information (coordinates), it is able to calculate the object's current location. The more satellites are used to find a location the more exact it is. Trilateration is also used today to locate the epicenter of earthquakes. When an earthquake occurs, by knowing the speed at and how far away it occurred from the earthquake, you can find the exact epicenter of the earthquake.

Astronomers are able to find the distance of earth from a star by applying the laws of sines, by which the astronomer find out the angle of elevation of earth to a star in opposite sides of its orbit. By knowing these angles, and the diameter of earths orbit, astronomer's are able to find the third angle, the star's distance from the sun, and with a simple application of the Pythagorean Theorem, astronomers are able to find earths distance from a star.

http://science.howstuffworks.com/question224.htm

For example, if an earthquake occurred 300 kilometers away from city X, 500 kilometers away from city Y, and 150 kilometers away from City Z, then you can find the epicenter at the point where all three circles cross over.

http://mattdonahoe.com/springs/



The Tortoise vs. Achilles

Zeno's paradox that will leave you thinking

Emilio Moreno

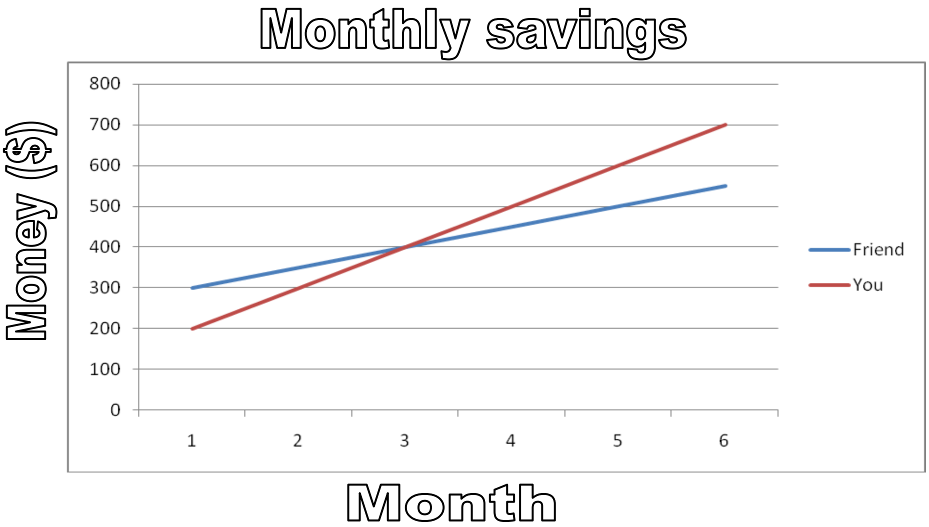
Zeno's paradox of "The Tortoise and Achilles" states that if the turtle is given head start before Achilles, it will win the race. It says that a man will run faster than the turtle, but before passing it, he must first reach the point where the turtle is. In the time he took to reach that point, no matter how fast he did, the turtle has covered some more distance. In other words every time Achilles wants to pass the turtle he must first reach the point where his competitor was, but in the time he took to do so, the turtle has advanced some more. According to Zeno's paradox, it would be impossible to pass an object in motion if they are competing in the same line and one of them already has a head start!

Achilles must first reach point B before passing the turtle. In the time interval Achilles took to do so, the turtle reached point C…

Achilles must now reach point C before passing the turtle. In the time interval Achilles took to do so, the turtle has now reached point D. This patter would repeat an infinite number of times, and, theoretically, Achilles would never win the race with the turtle.

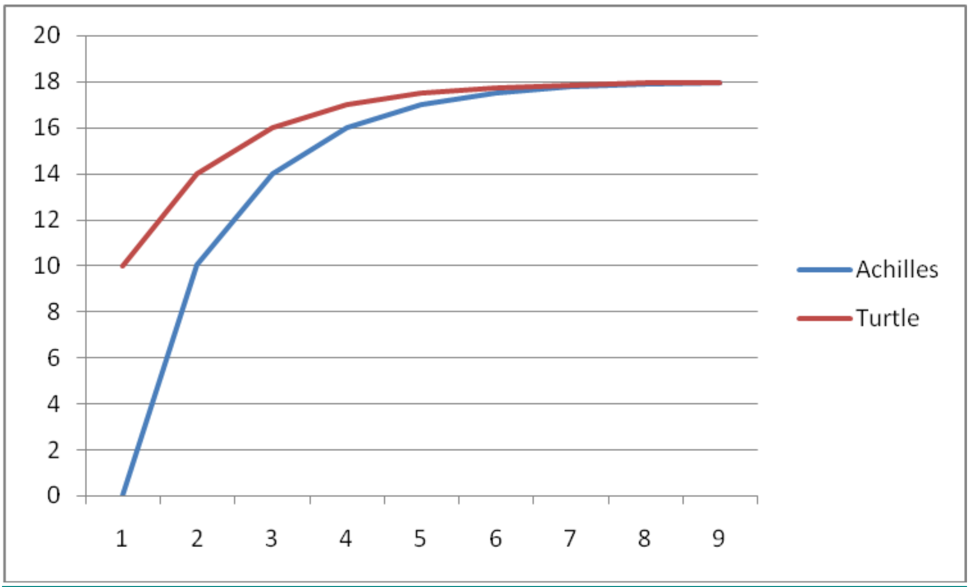
The explanation to this apparent paradox is fairly simple. Achilles cannot pass the tortoise if they both stop every time Achilles reaches the point where the tortoise was located. Contrarily in a situation in which they both keep on going without stopping, Achilles would win the race because he is faster than the tortoise. Another example for this is the following:

Consider that your friend opens a bank account 1 month before you do and starts with $300, but he adds $50 to his account every month, while you began your account with only $200, but add $100 every month. At the end of the first month, you would have $300 saved up and your friend would have $350. You have already reached the quantity your friend previously had (like Achilles reaching the point where the tortoise was). Next month, you both have $400 in the bank, but at the end of the third month, you would have saved up $500, while your friend would only have $450. This proves that this paradox only seems true because of the wording used in it and that such situation would only occur if the person behind the other one would stop every time they reach the previous point of location of the one that is ahead.



	You	Your Friend
Month 1	$200	$300
Month 2	$300	$350
Month 3	$400	$400
Month 4	$500	$450
Month 5	$600	$500
Month 6	$700	$550
Month 7	$800	$600
Month 8	$900	$650

But we know Achilles could beat the turtle…

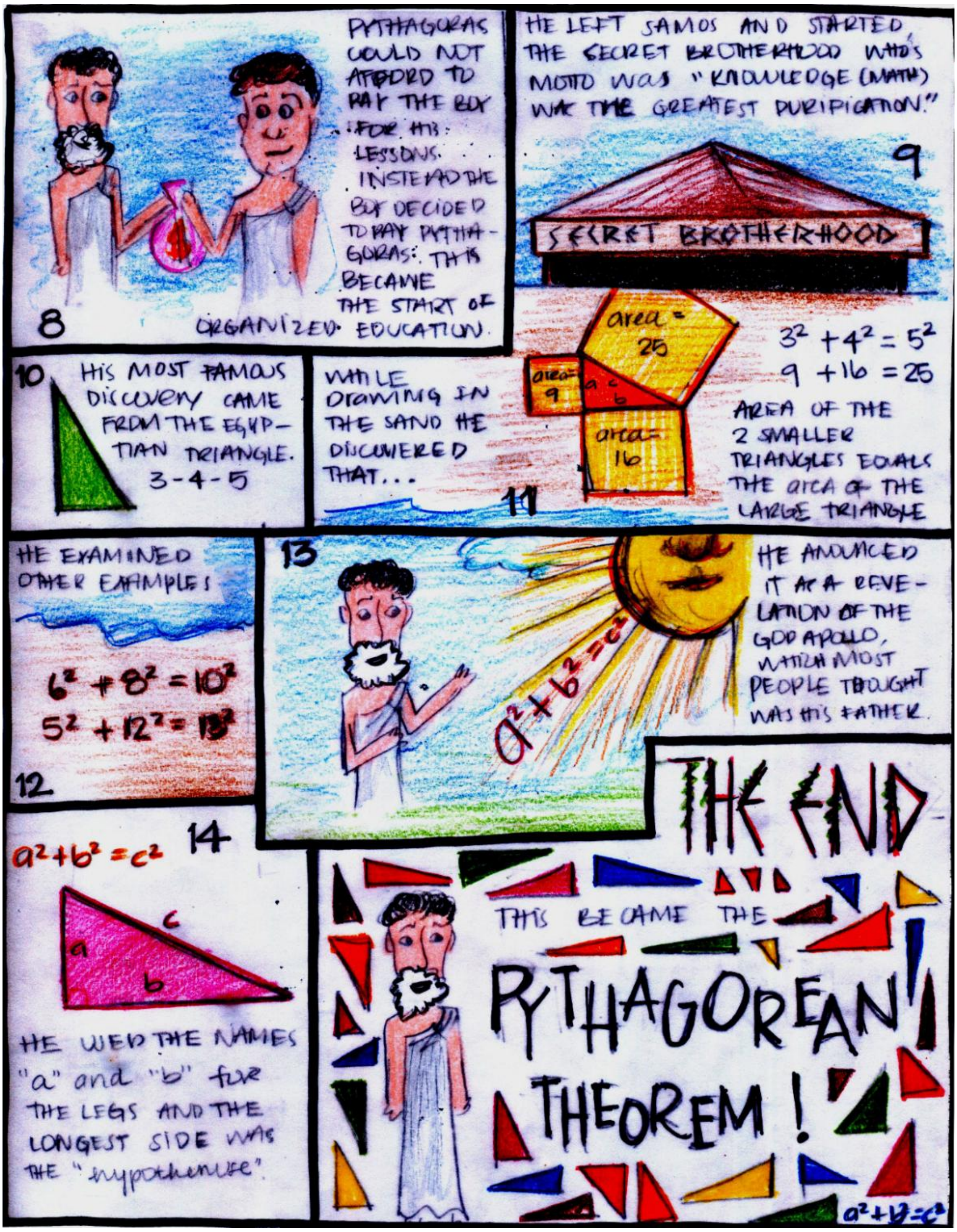


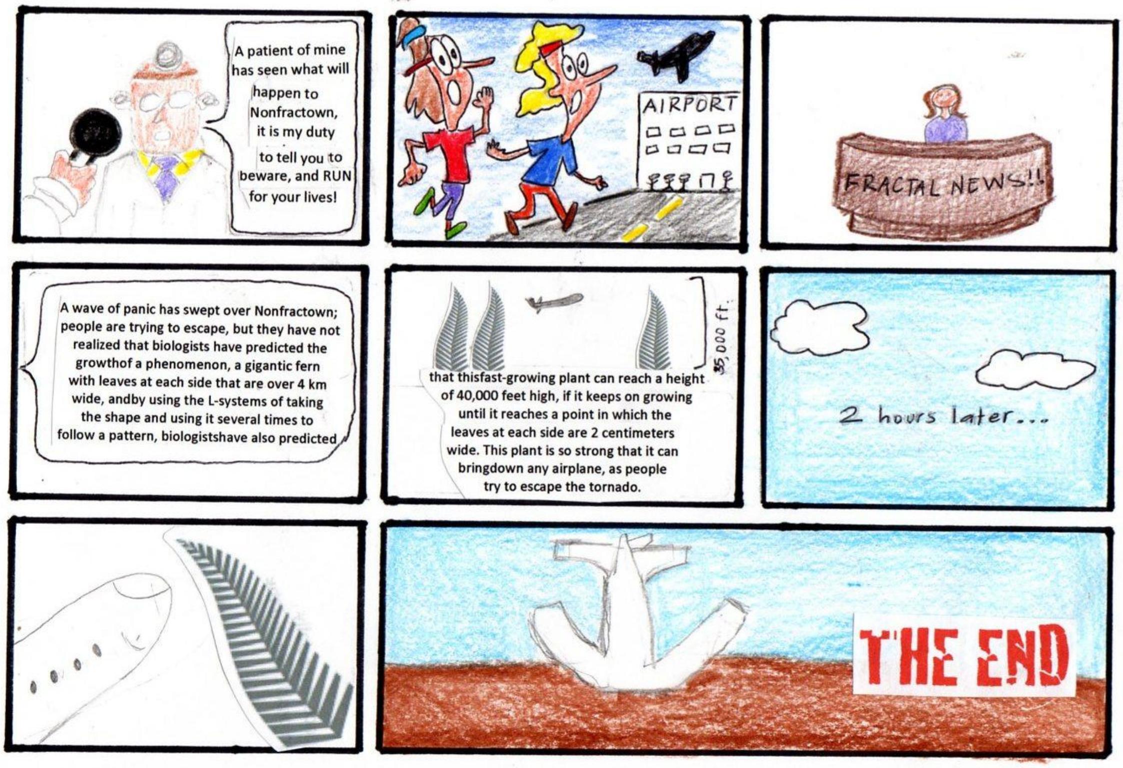
This is a graph showing the situation presented in the paradox. This paradox would only be true if Achilles stopped every time he reaches the point where the turtle previously was. The graph shows that the velocity would not be constant if the paradox was true.

Source: http://www.mathacademy.com/pr/prime/articles/zeno_tort/index.asp

Comics

Continued From Edition 1





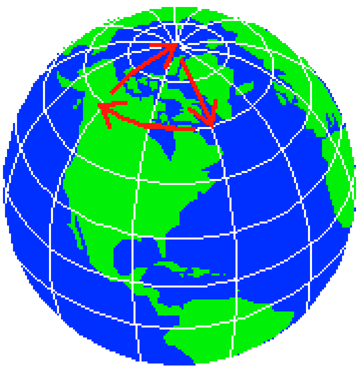
Brain Teasers Answers

All Credits go to http://www.scientificpsychic.com/mind/mind1.html

1.) Was you answer 18?

2.) The bear was white because it was a polar bear. The only place on earth where a bear can go south, west and north equal distances and end up where it started is the North Pole.

Actually, the bear could go west two or five kilometers instead of one and it would not make any difference -- the bear would be making a *circle* around the North Pole. East and West you travel along parallels which are circles equidistant from the poles. North and South you travel along meridians which are circles that cross both the north and the south poles.



Creating an Illusion of Depth

Perspective, developed during the 1500's, is a method that is used to give the illusion of depth onto a flat surface.

What is Perspective?

Perspective is a theory of a drawing, which allows artists to represent graphically three-dimensional objects on a flat surface or plane. This technique gives the illusion of depth onto a flat surface. Geometrically, it is representing on paper, on which objects appear to get smaller and closer together. This type of perspective begins with a line that defines the farthest distance of the background and the vanishing point.

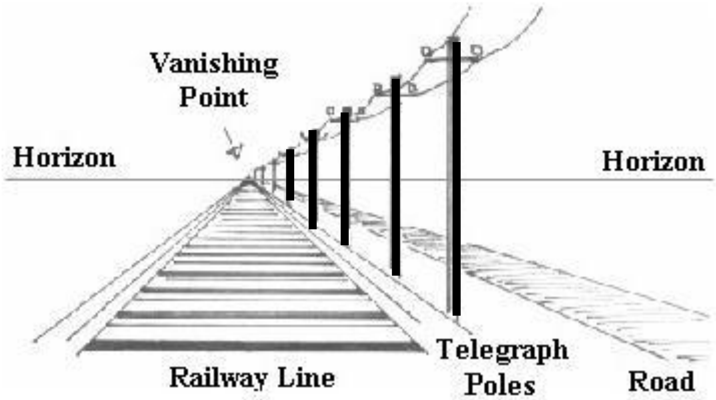
Types of Perspective:

There are four main types of perspective used in drawing. They all have the same concepts but the numbers of vanishing points vary.

One-Point Perspective:

One-Point perspective is when all major lines of an image come together at one point. Also, this type of perspective is present when both the height and width of an object are parallel to the picture plane. This type of perspective has a single vanishing point. The vanishing point is the point in which parallel lines seem to come together and disappear. The lines that are drawn are parallel to each other, but when applying one-point perspective, the lines seem to meet and come together in one point. One-point perspective images draw the viewer along the lines to the vanishing point.

http://www.animationbrain.com/one-point-perspective.html

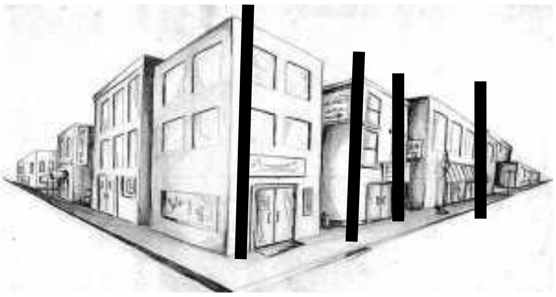


An example of one point perspective is the set of straight railroad tracks or a road. The lines are the same distance apart by they seem to meet and join in the distance.

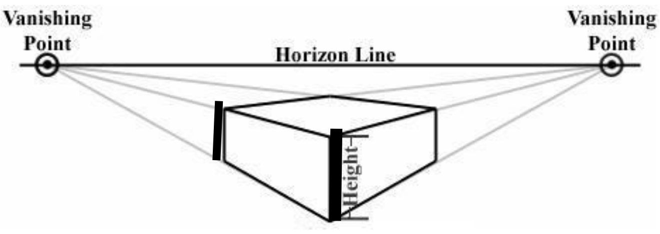
Two-Point Perspective:

Another type of perspective is the two-point perspective. This type of perspective contains two vanishing points on the horizon line. Two- point perspective is when only the height is parallel to the picture plane. The farther apart the vanishing points are located from each other, the more we can see of the sides and the closer the points are, the lesser we can see.

The main difference between one-point perspective and two point perspective is the number of vanishing points on the horizon line.



http://kingfishers.ednet.ns.ca/art/grade10/drawing/perspective3.html



http://www.homeschoolarts.com/per-l1-3.htm

Geometrically this is how perspective drawing is done.

Three-Point Perspective:

In this type of perspective things become more challenging. Every line will converge on one of the three points.The main difference between the three and the two-point perspective is that the three-point perspective contains another vanishing point located above or below the horizon line. When the vanishing point is located near the object, the object looks bigger.
Both are examples of three-point perspective drawing where here are three vanishing points.

http://www.jeffprentice.net/teachf/onlineperspective.htm

http://www.homeschoolarts.com/per-l1-4.htm

http://ed101.bu.edu/StudentDoc/current/ED101fa09/rdubovy/zeropoint.html

Zero- Point Perspective:

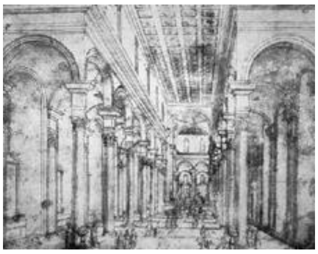
Zero-Point perspective is applied when there is no vanishing point. In this type of perspective, all the lengths sides, width sides, and height sides are all parallel. Artists use perspective to give the illusion of depth without using vanishing points. Artists draw the objects that are closer to the viewer larger and with more detail and the objects that are father away smaller and with less detail. Three-point perspective only occurs when there is no linear perspective.

http://ed101.bu.edu/StudentDoc/current/ED101fa09/rdubovy/zeropoint.html

Most drawings in zero-point perspective are nature drawings.

History

Perspective has been used since the 1400's, in Florence Italy, several renaissance artists have applied this geometric method in their famous art pieces. Filippo Brunelleschi, a Florentine architect, first developed perspective. His goal was to make his paintings in order to show a much better representation of reality. Brunelleschi demonstrated this method by taking some buildings found in Florence, and painting their outlines into a mirror. With this experiment he was able to prove that all lines converged on the horizon line.On the pictures below one, can clearly see how certain artists applied perspective in their paintings and drawings.



http://www.explore-drawing-and-painting.com/perspective-drawing.html

http://www.internal.schools.net.au/edu/lesson_ideas/renaissance/renaissance_perspective.html

http://www.internal.schools.net.au/edu/lesson_ideas/renaissance/renaissance_perspective.html

This is a sketch of the Church of Santo Spiritu, the artist was Brunelleschi.

Parallel Lines

Parallel lines, are lines that exist on the same plane and never intersect. Having the same slope is another characteristic. In perspective drawing, parallel lines intersect at the horizon, the point were this two lines meet is called a vanishing point. In real life, railroad lines never intersect, therefore are parallel. However when drawn they intersect on the horizon, in order to create depth.

"*Perspective is to painting what the bridle is to the horse, the rudder to a ship……………..There are three aspects to perspective. The first has to do with how the size of objects seems to diminish according to distance: the second, the manner in which colors change the farther away they are from the eye; the third defines how objects ought to be finished less carefully the farther away they are.*" (Leonardo Vinci)

http://www.artyfactory.com/perspective_drawing/perspective_index.htm
http://www.webexhibits.org/sciartperspective/raphaelperspective1.html
http://en.wikipedia.org/wiki/Perspective_(graphical)
http://www.animationbrain.com/history-of-perspective.html
http://psych.hanover.edu/krantz/art/linear.html
http://www.explore-drawing-and-painting.com/perspective-drawing.html
http://www.renaissanceconnection.org/lesson_art_perspective.html
http://www.homeschoolarts.com/per-l1-4.htm