Note Title 3/15/2007

Advanced Measurement Techniques

1

Signals Buried in Noise

- Typically convert system outputs to electronic signals
- In terms of voltages

$$V(t) = V_0 + V_{noise}$$

- If noise is well behaved

$$\langle V_{noise} \rangle = 0$$

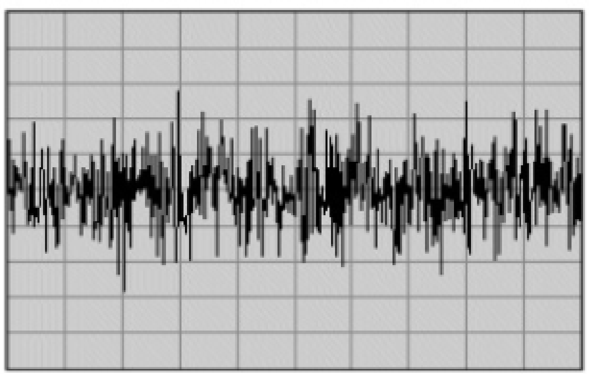
- By averaging

$$\langle V(t) \rangle = \langle V_0 + V_{noise}(t) \rangle = \langle V_0 \rangle + \langle V_{noise} \rangle \rightarrow V_0$$

If $\langle V_{noise} \rangle \neq 0$ we have systematic error

<u>White Noise</u>

– sometimes called Gaussian noise



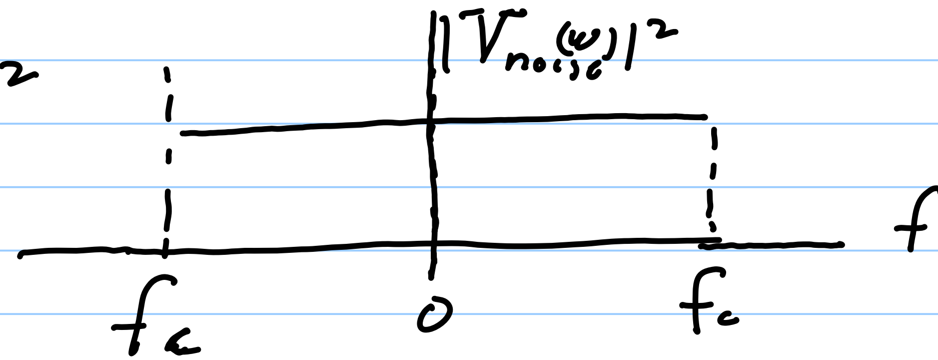
White Noise

Statistics of noise independent of time provided $|t-t'| > \tau_c$

correlation time

Power spectrum of white noise

$$\left|\tilde{V}_{noise}(\omega)\right|^2$$

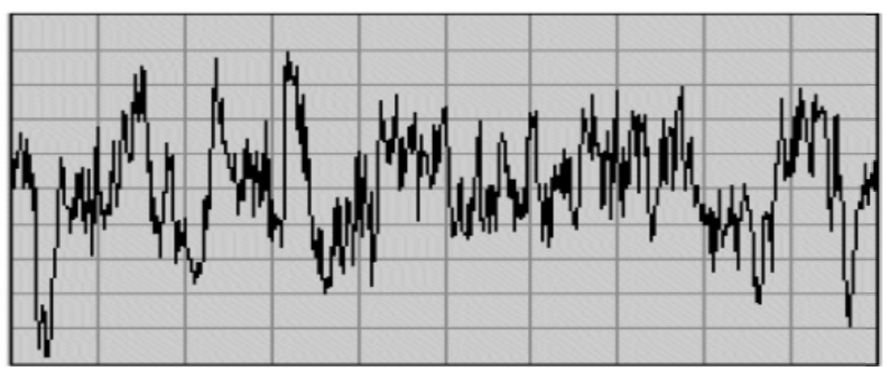


$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{\tau_c}$$

- In the real world noise is seldom white!
- Amplifiers and other noise generating element often drift slowly with time

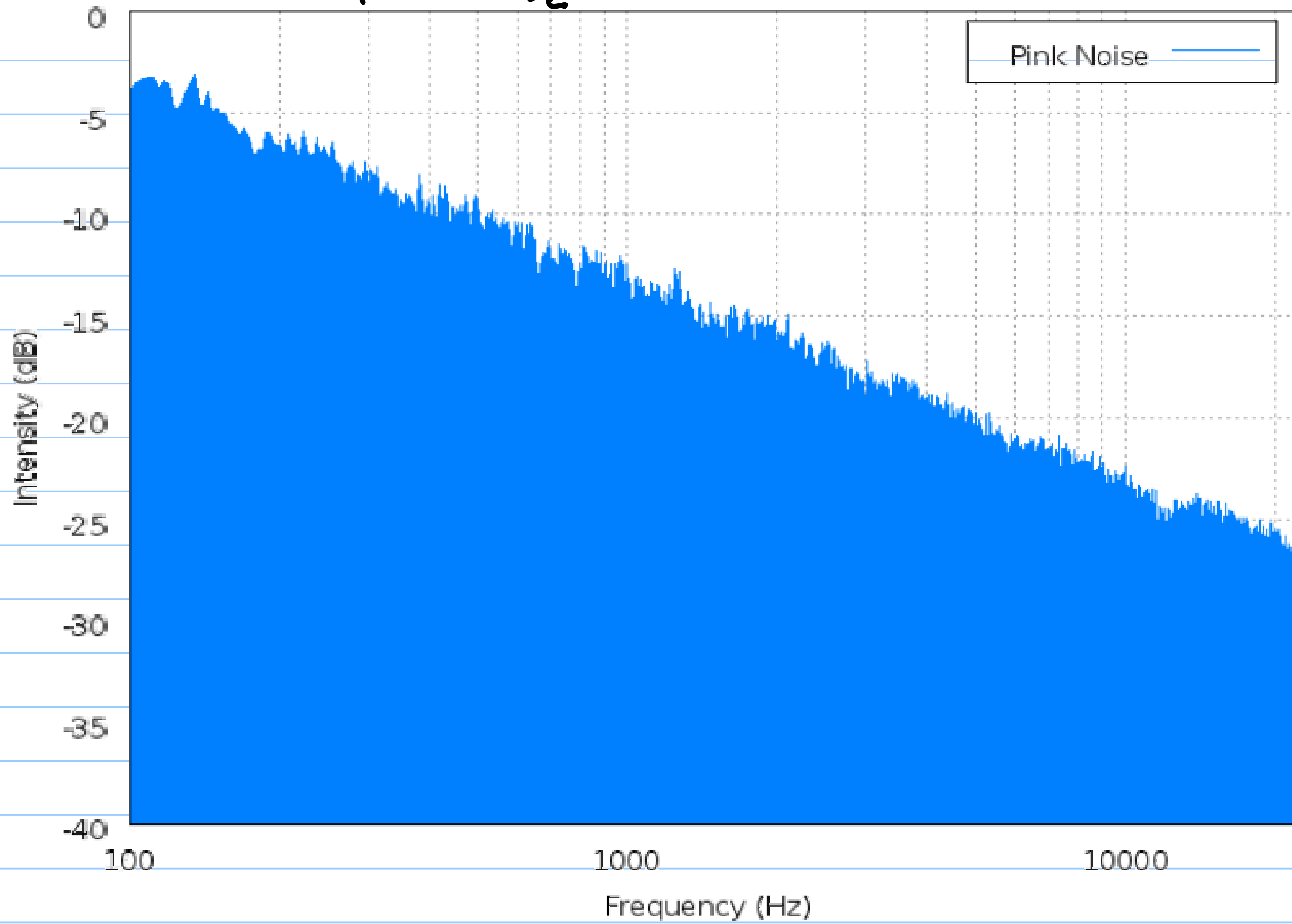
"$1/f$ noise"

- detailed studies of many different types of noise have shown the freq. spectrum goes approximately as $1/f$ where f is the frequency hence "$1/f$ noise"

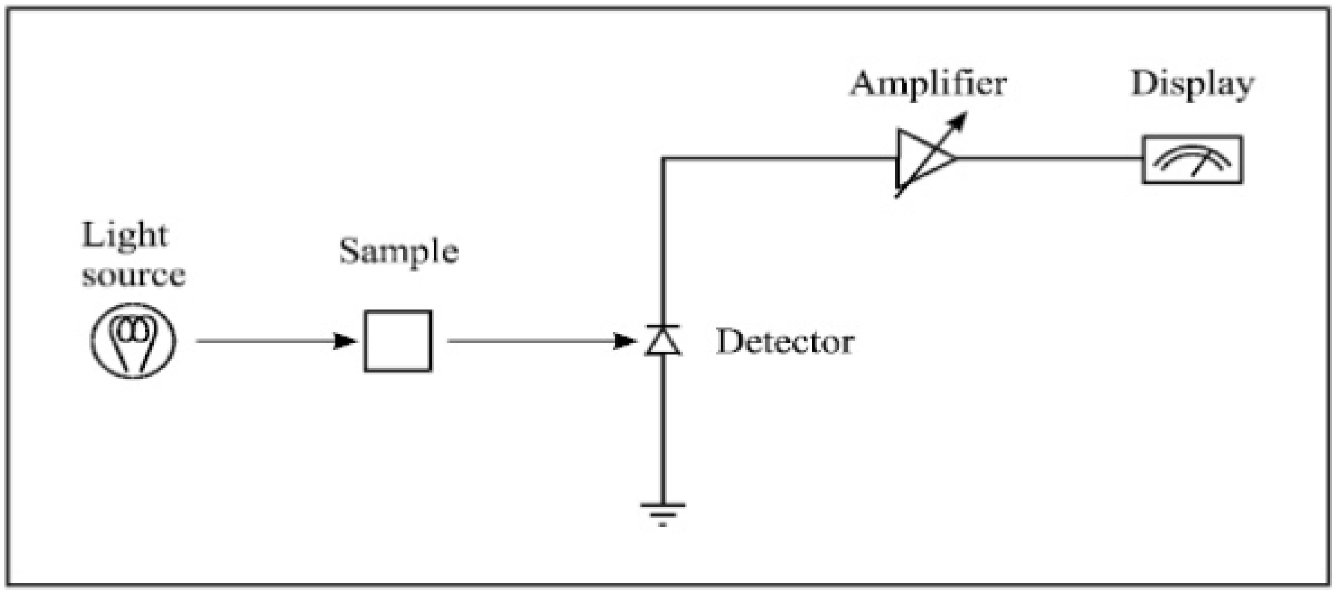


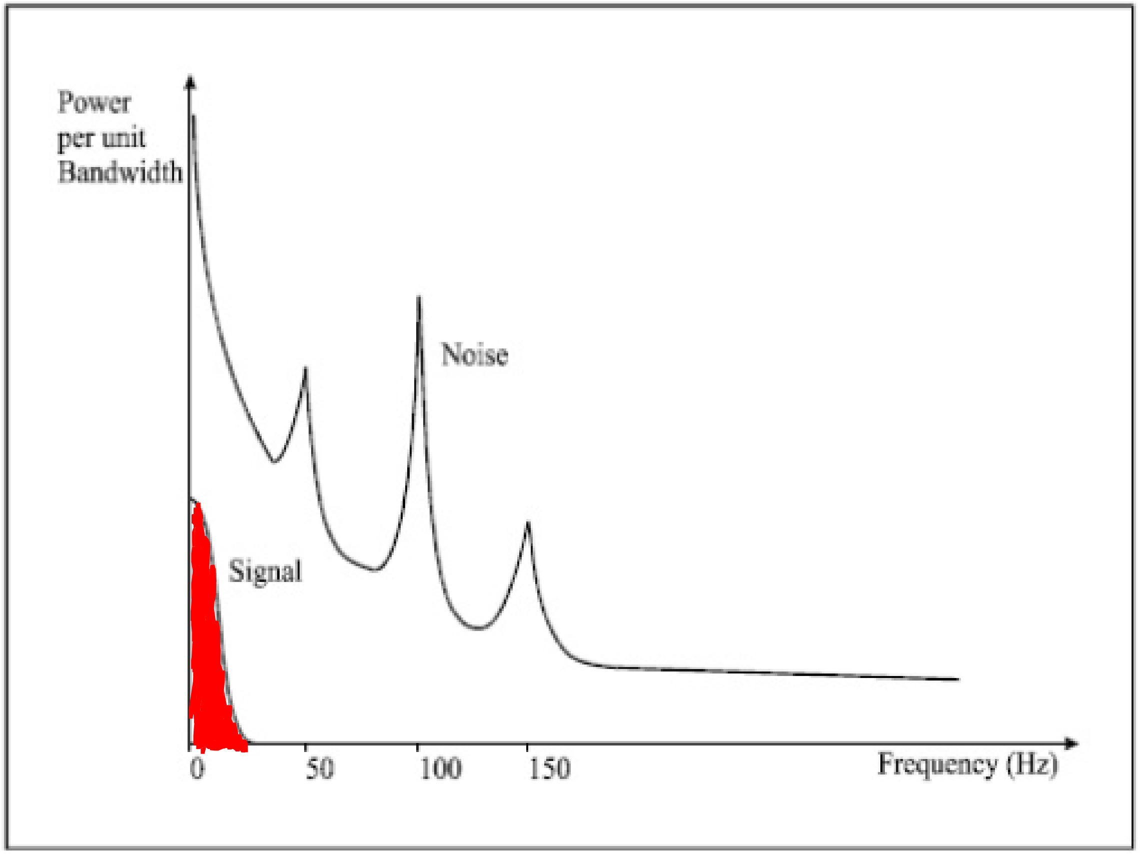
1/f Noise

$$\left| \bar{V}_{noise}(\omega) \right|^2$$



Typical Experimental Setup




Power
per unit
Bandwidth
Noise
Signal
0
50
100
150
Frequency (Hz)

In addition most detectors have an unwanted DC offset

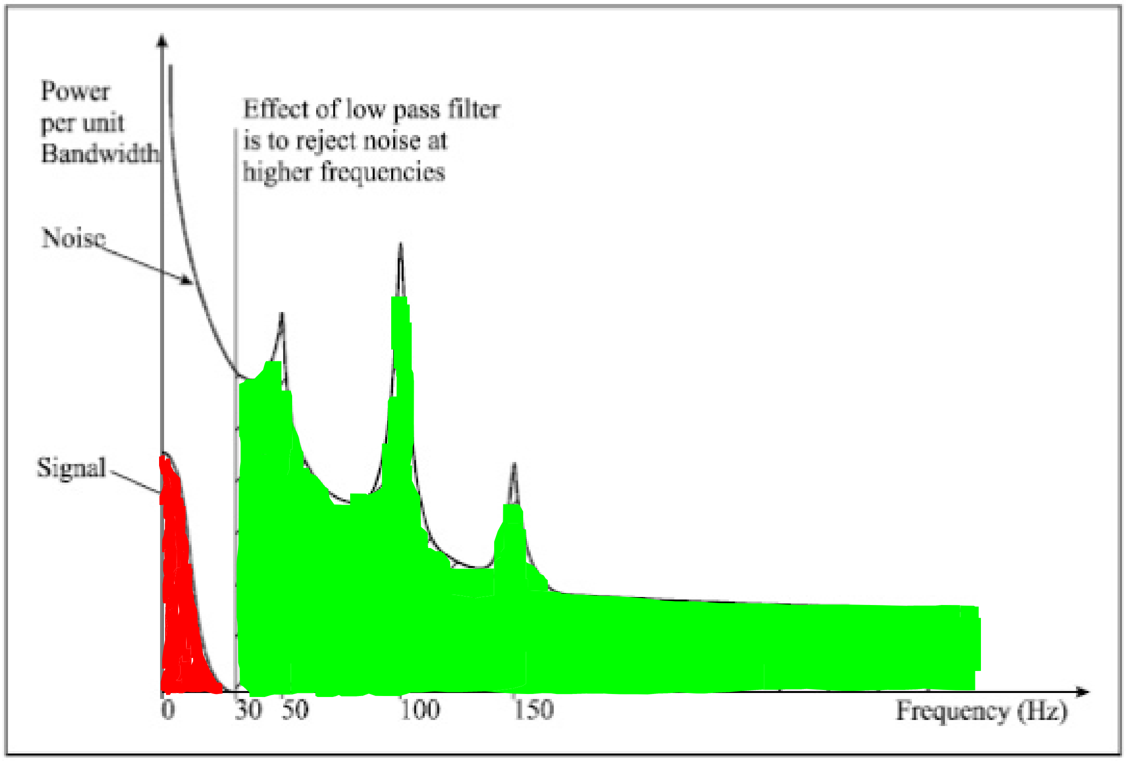
Thus the signal voltage becomes

$$V(t) = V_0 + V_{offset} + V_{white\,noise}(t) + V_{1/f\,noise}$$

If dominated by 1/f noise averaging will not help to improve determination V_0

What to do?

Effect of low pass filter



Spectrum with Bandpass Filter

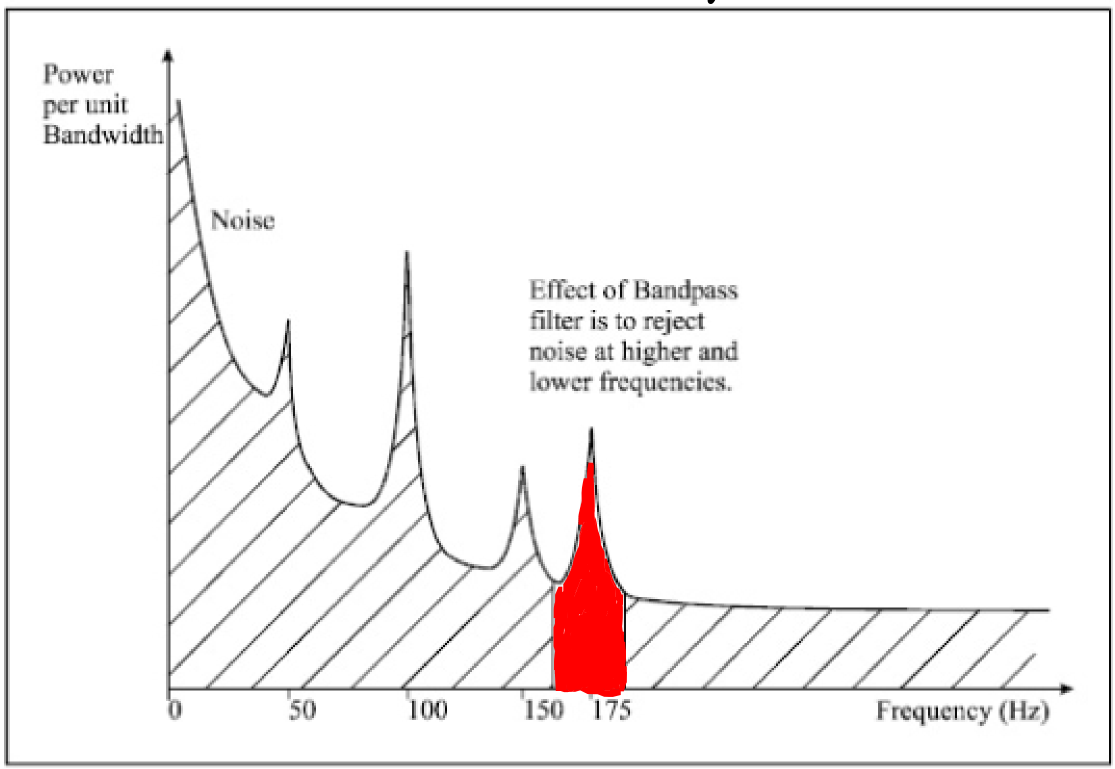
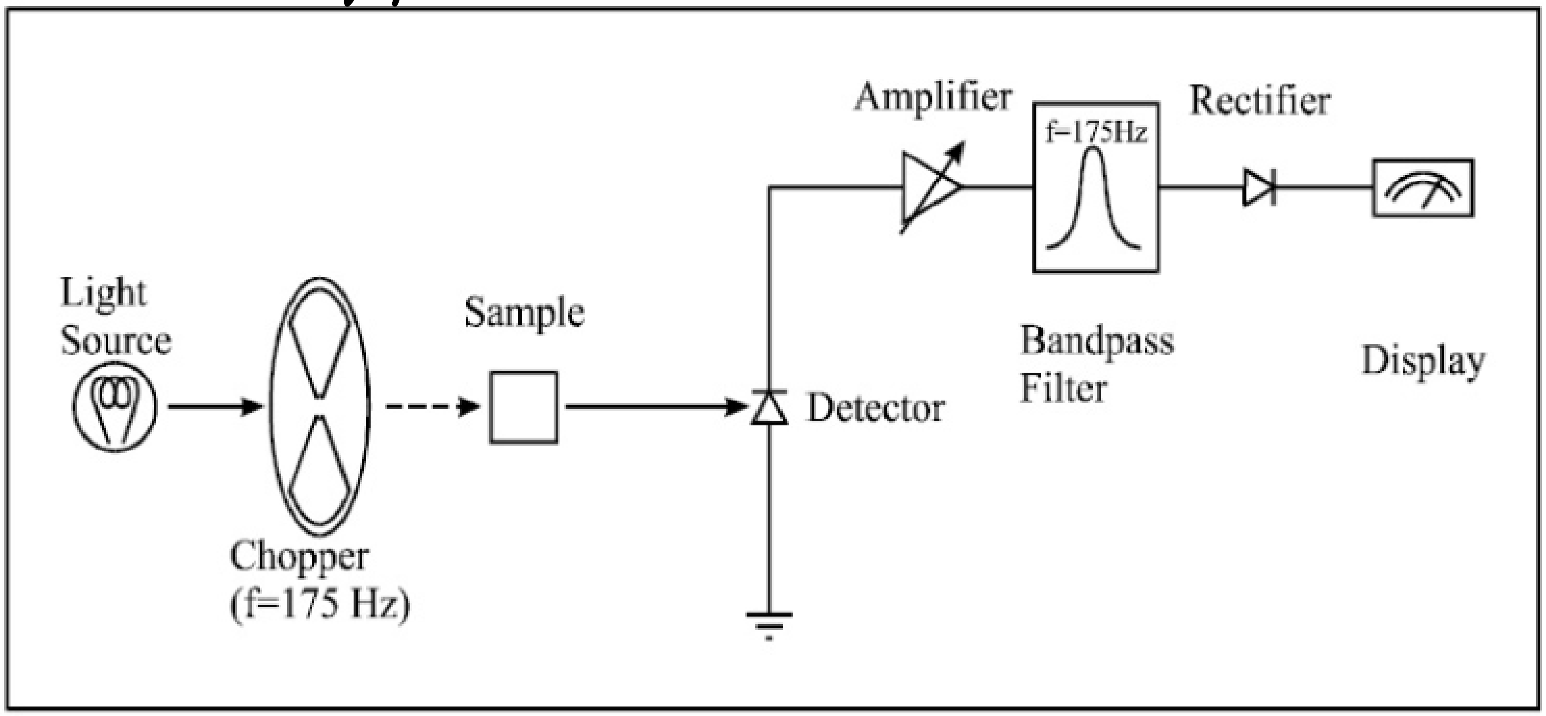


Fig 4 Effect of Bandpass Filter at 175 Hz.

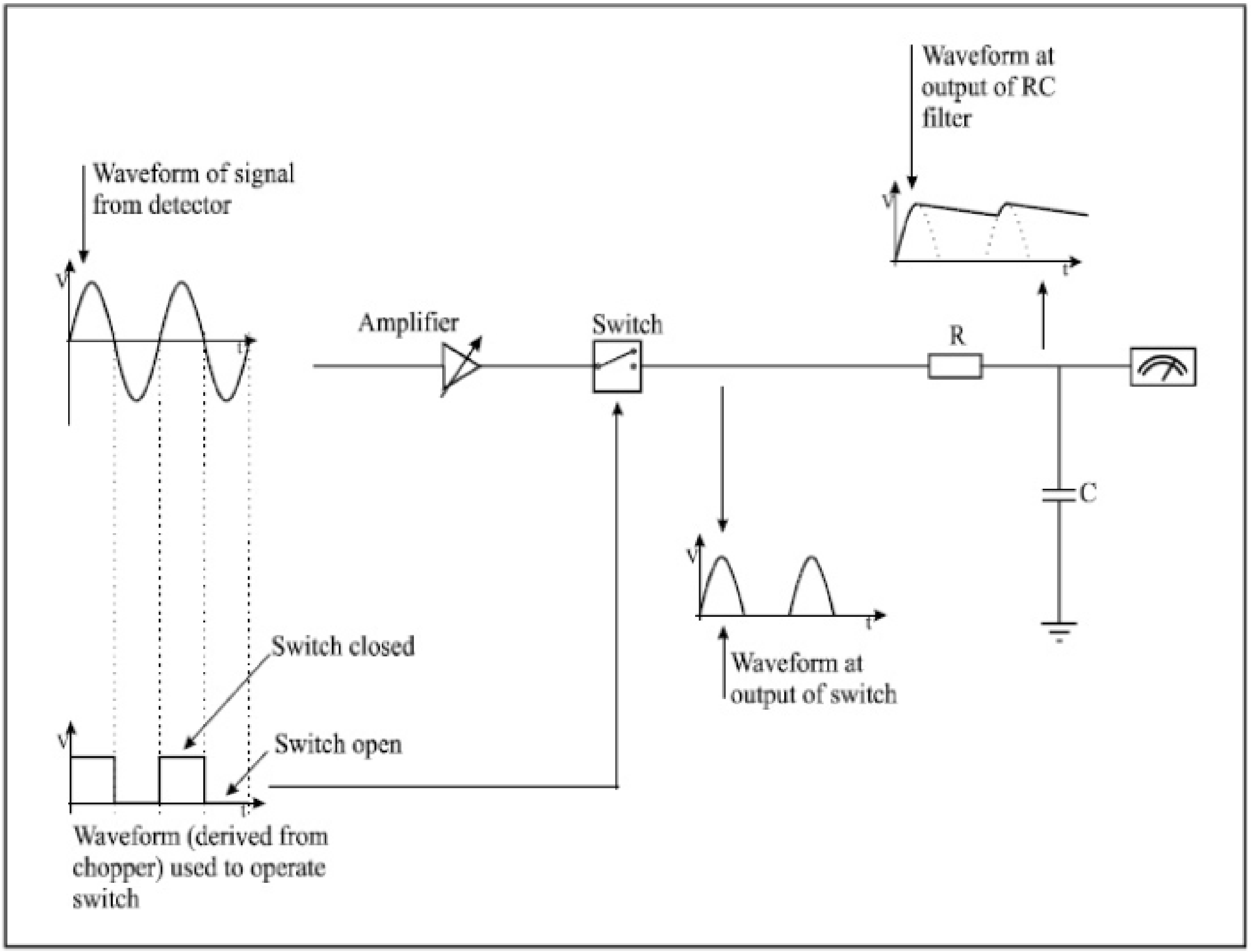
- Trick: Chop the signal at high frequency and measure difference $V_{on} - V_{off}$
- Averaging then concerns only noise at the chopping frequency

- 4 Hz to 3.7 kHz chopping frequencies
- Low phase jitter
- Single and dual beam experiments
- Sum & difference reference outputs
- Bolt clamp or rod mounting
- SR540 ... $1095 (U.S. list)

Chopper Set Up



Phase Sensitive Detector



The reference signal

- converted to a sinewave with phase
- multiplied by signal

Thus

$$V_{out} = \langle V_{sig} \cos(\omega_R t + \phi) \rangle$$

running average

When ϕ chosen properly

$$V_{out} = \alpha V_0$$

Dual-phase lock in

- in-phase

$$V_x = \langle V_{sig} \cos(\omega t + \phi)$$

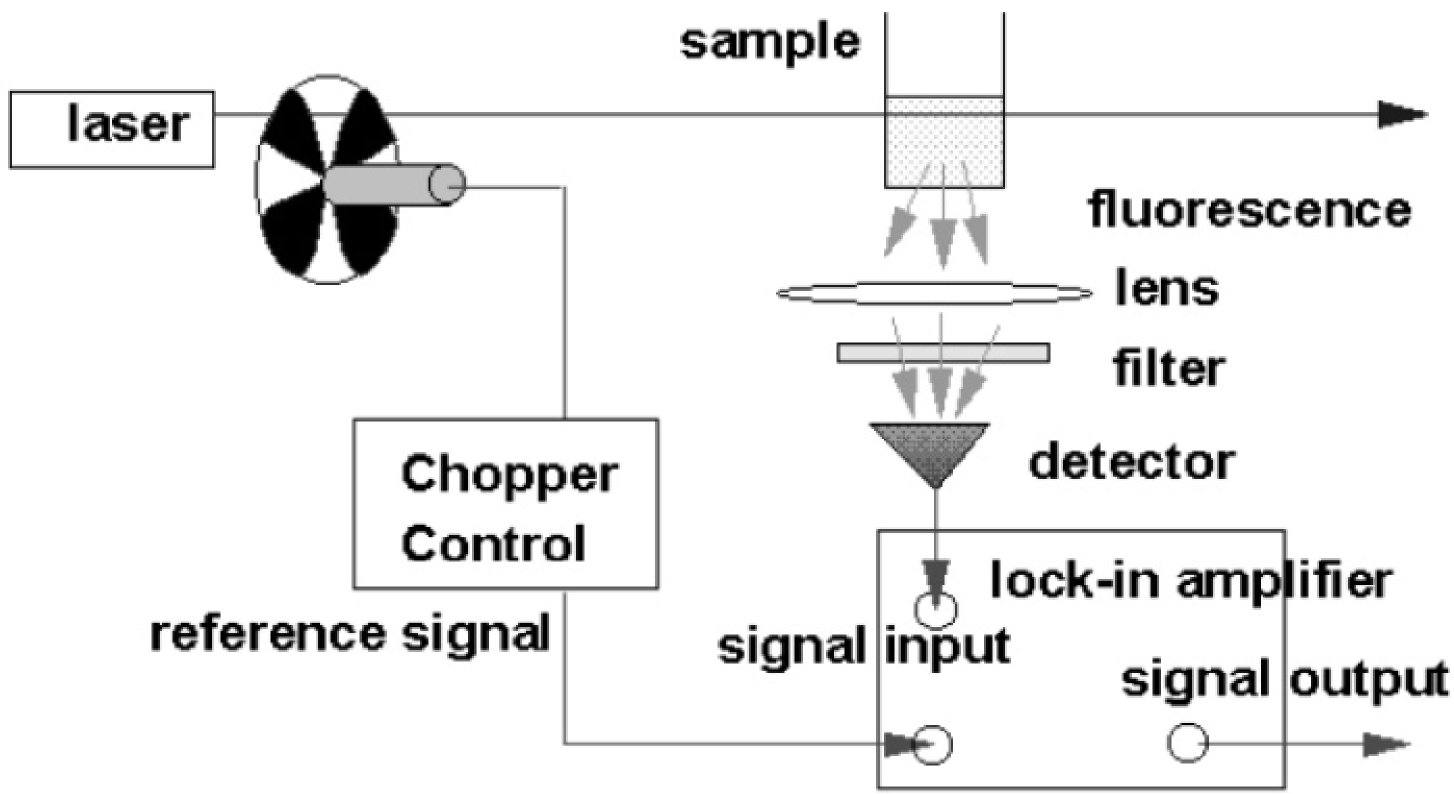
- quadrature

$$V_y = \langle V_{sig} \sin(\omega t + \phi)$$

Yielding

$$V_R = \left(V_x^2 + V_y^2\right)^{1/2}$$

$$\phi = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$


laser
sample
fluorescence
lens
filter
detector
Chopper Control
reference signal
lock-in amplifier
signal input
signal output

Noise Spectral Density

$$V_{noise,\tau}(t) = \frac{1}{\tau}\int_{t'=t}^{t+\tau} V_{noise}(t')\,dt'$$

Over some averaging time T_{ave}

For white noise the noise power spectrum becomes

$$P_{noise,B}(f) = \left| \frac{1}{T_{ave}} \int V_{Noise}(t)\, e^{i2\pi f t}\, dt \right|^2$$

$$P_{noise,B}(f) = \sigma_{noise,\tau}^{2}\,\tau B$$

where

$$\langle V_{noise,\tau}(t)^2 \rangle^{1/2} = \sigma_{noise,\tau} \sim \tau^{-1/2} \text{ for white noise}$$

$$B = \frac{1}{\tau_{ave}}$$

Bandwidth independent power spectral density

$$S(f) = \lim_{\tau_{ave}\to\infty} \frac{1}{\tau_{ave}} \left| \int V_{noise}(t)\, e^{i 2\pi f t}\, dt \right|^2$$

$$S(f) \approx P_{noise,B}(f) \Big/ B$$

For white noise

$$S(f)^{1/2} = \text{constant}$$

For $1/f$ noise

$$S(f)^{1/2} \sim \frac{1}{f}$$

Note that

$$B = \frac{\beta}{T_{filter}}$$

Noise in lockin $\sim T_{filter}^{-1/2}$

Thermal noise

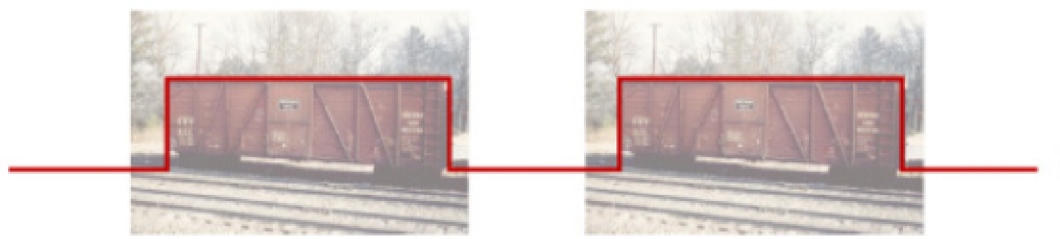
$$V_{RMS\ thermal} = \sqrt{4kTRB}$$

where R = resistance
B = bandwidth

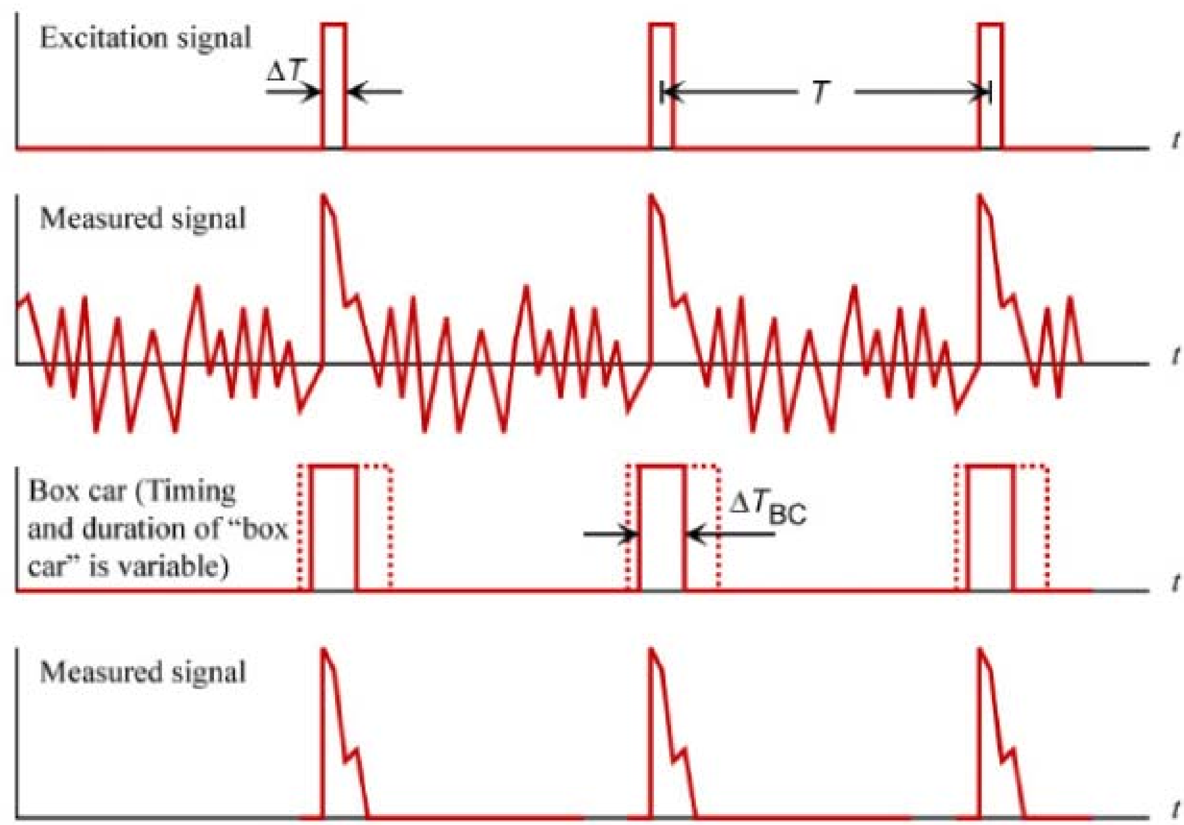
Box-car integrator

- The box-car integrator is used for the detection of noisy pulsed signals.
- Box-car integrator enables *gated detection*.
- What is a box car?

- Why a box-car integrator is called that way?



- Typical signals:



- Excitation signal is pulsed.
- Measured signal is noisy. However, during the times of excitation, the signal/noise ratio improves.
- Box car integrator integrates over a number of pulsed excitation events.

Signal Improvement With BoxCar Integrator

$$V = \frac{1}{N}\sum_{\text{pulse } i=1}^{N} V_{\text{signal},i} + \frac{1}{N}\sqrt{\sum_{\text{pulse } i=1}^{N} V_{\text{noise}}^2} = V_{\text{signal}} + \frac{1}{\sqrt{N}}\sqrt{\overline{V_{\text{noise}}^2}} .$$

Thus the signal/noise ratio after N measurements is given by

$$\frac{V_{\text{signal}}}{V_{\text{noise}}} = \frac{V_{\text{signal}}}{\frac{1}{\sqrt{N}}\sqrt{\overline{V_{\text{noise}}^2}}} \propto \sqrt{N} .$$

- The box-car integrator improves the signal/noise ratio by

  (i) ***Gating the detection***. What is the improvement in signal/noise ratio attainable by gating?

  (ii) ***Averaging over multiple pulses***. An improvement in the signal/noise ratio is attained by averaging over N pulses. What is the magnitude of the improvement?

Time Correlated Photon Counting

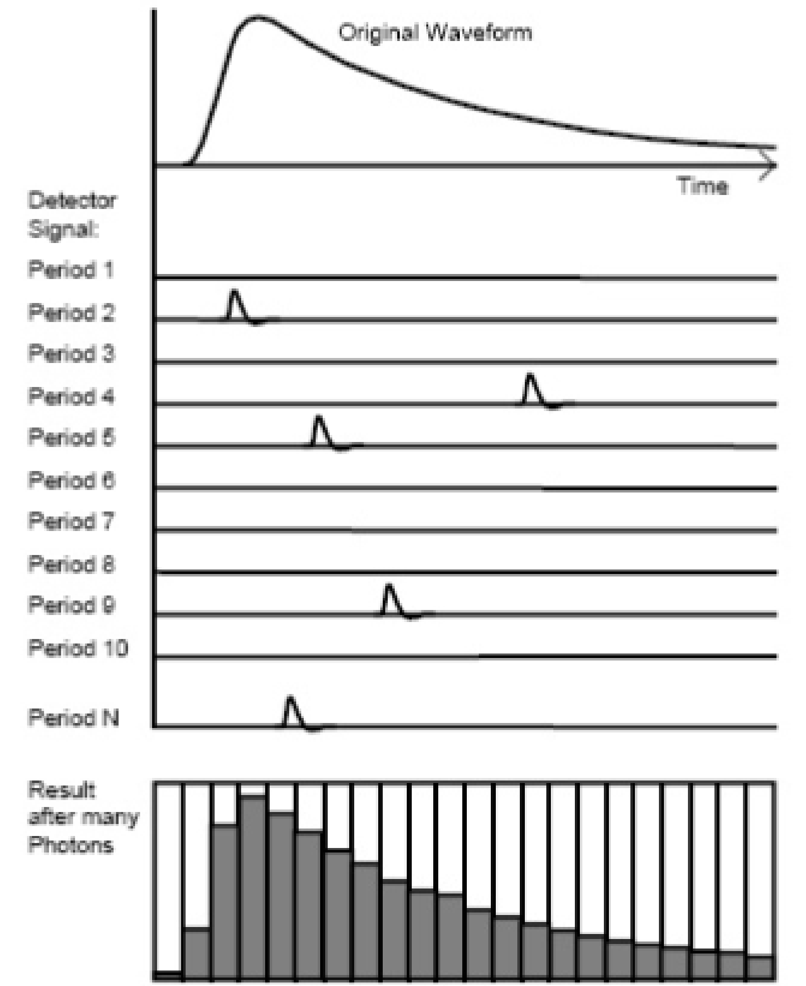
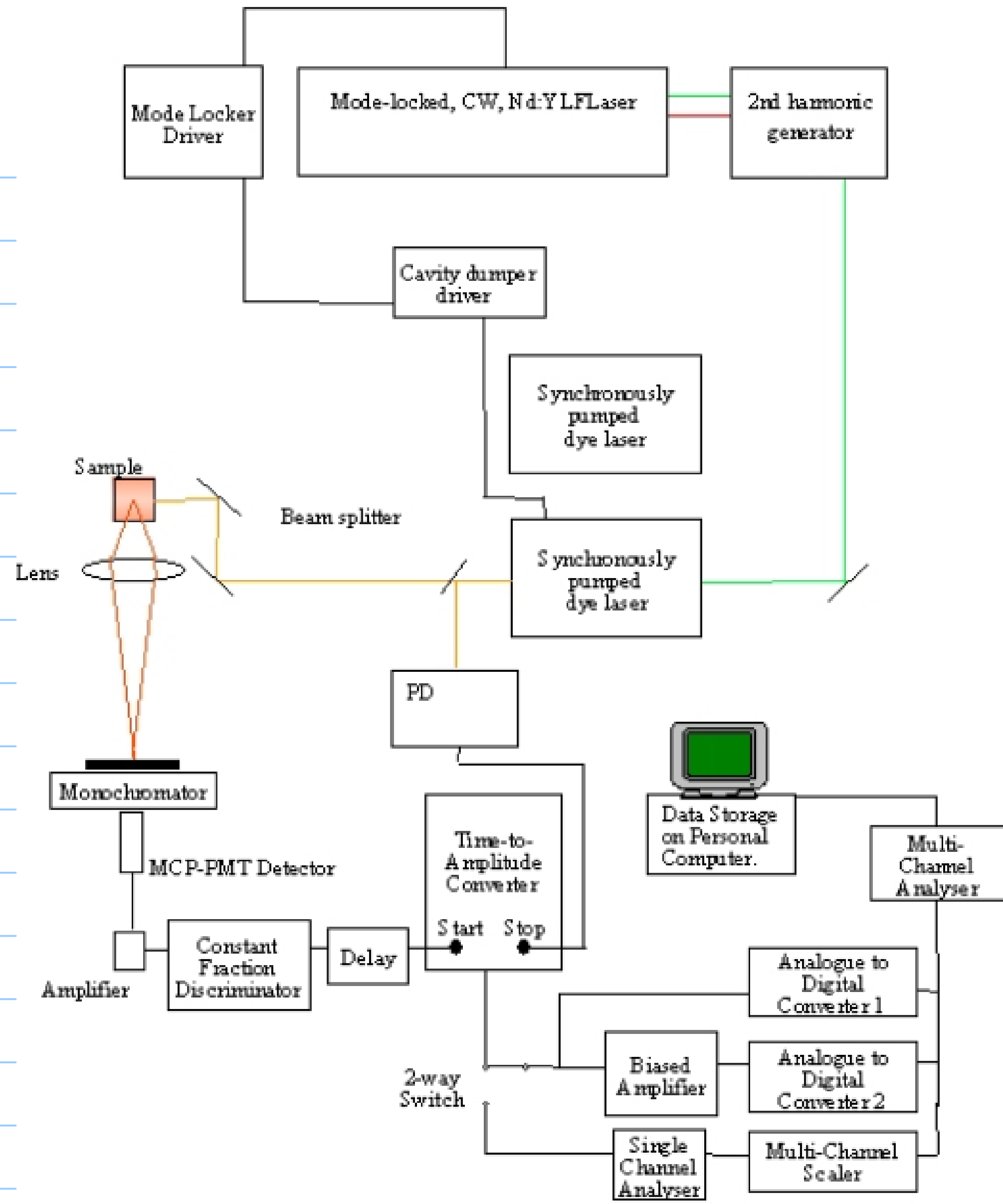


Fig. 1: TCSPC Measurement Principle


Mode Locker Driver
Mode-locked, CW, Nd:YLFLaser
2nd harmonic generator
Cavity dumper driver
Synchronously pumped dye laser
Synchronously pumped dye laser
Sample
Beam splitter
Lens
PD
Monochromator
MCP-PMT Detector
Amplifier
Constant Fraction Discriminator
Delay
Time-to-Amplitude Converter
Start
Stop
Data Storage on Personal Computer.
Multi-Channel Analyser
Analogue to Digital Converter 1
Biased Amplifier
Analogue to Digital Converter 2
2-way Switch
Single Channel Analyser
Multi-Channel Scaler