natural quartz crystals is similar to that predicted by the dynamical theory for perfect, absorbing crystals.

Using the results of the theoretical investigation, it has been possible to obtain experimental $F$ values with an accuracy of $\pm 10 \%$ from one crystal, without the necessity for making detailed assumptions about the degree of perfection of its texture. From the combination of observations from a series of crystals it may be possible to increase the accuracy of this method. Higher accuracy may be obtained if the absorbing effect of any projections on the crystal surface can be found from the experimental variation of $J(\beta)$ with $\beta$ for a weak reflexion.
A disadvantage of the method is that the crystals used must be quite large, and show faces suitable for mounting. However, wherever applicable the method leads to far more accurate results than any previous investigation.

It is a pleasure to thank Prof. Sir Lawrence Bragg and Dr W. H. Taylor, in whose laboratory this work was carried out, for their constant help and encouragement. I am indebted to Dr S. O. Agrell, Curator of the Museum of the Department of Mineralogy and Petrology, Cambridge, for the provision of the two quartz crystals. I would also like to thank my colleagues, Dr P. B. Hirsch and Dr
J.S. Thorp, for their invaluable help and advice throughout this work. This research was carried out while the author was in receipt of a maintenance grant from the Department of Scientific and Industrial Research.

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# The Structure of Urea. Interatomic Distances and Resonance in Urea and Related Compounds* 

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(Received 7 January 1952)


#### Abstract

The interatomic distances in urea have been redetermined by Fourier and least-squares analysis of complete data obtained with $\mathrm{Cu} K \alpha$ radiation. The results are $\mathrm{C}=\mathrm{O}=1.26 \AA, \mathrm{C}-\mathrm{N}=1.34 \AA$, $\angle \mathrm{N}-\mathrm{C}-\mathrm{O}=121^{\circ}, \mathrm{N}-\mathrm{H} \cdots \mathrm{O}=2.99 \AA$ and $\mathrm{N}-\mathrm{H} \cdots \mathrm{O}^{\prime}=3.04 \AA$. Evidence for the position of the hydrogen atoms is found in the electron-density sections. Improved agreement of calculated with observed structure factors was obtained by including the hydrogen scattering in the calculations, and by use of anisotropic temperature factors.

Revised values for carbon-oxygen and carbon-nitrogen double bonds are suggested, and these values are used to discuss the effect of resonance on interatomic distances in urea and related compounds.


## Introduction

Because of its simplicity, the structure of urea was one of the first organic structures to be determined

[^0]by X-ray crystallographic methods (Mark \& Weissenberg, 1923; Hendricks, 1928; Wyckoff, 1930, 1932; Wyckoff \& Corey, 1934). These early studies agreed on the space group, unit cell, and approximate parameters. The space group is $D_{2 d}^{3}-P \overline{4} 2_{1} m$; the unit cell contains two molecules of $\mathrm{CO}\left(\mathrm{NH}_{2}\right)_{2}$, with atoms in the following positions: 2 C in $(c):\left(0, \frac{1}{2}, z\right),\left(\frac{1}{2}, 0, \bar{z}\right)$; 20 in a second set of positions (c); 4 N in (e):
$\left(x, \frac{1}{2}+x, z\right),\left(\bar{x}, \frac{1}{2}-x, z\right),\left(\frac{1}{2}+x, \bar{x}, \bar{z}\right),\left(\frac{1}{2}-x, x, \bar{z}\right)$; and 8 H in two additional sets of positions (e). This structure is illustrated in Fig. l. The packing of the


Fig. 1. View of the urea structure, showing final bond distances and angles. Hydrogen bonds are indicated by the broken lines.
urea molecules is determined largely by hydrogen bonds, $\mathrm{N}-\mathrm{H} \cdots \mathrm{O}$.

A part of the attack in this laboratory on the problem of the structure of proteins has included the precise determination of the molecular dimensions and hydrogen bonding in various amino acids and related compounds. This made it seem worthwhile to determine, with the advanced techniques now available, the interatomic distances in crystalline urea in order to obtain additional precise information regarding the dimension of molecules involving resonance associated with the $-\mathrm{NH}-\mathrm{CO}$-group. A comparison of our final results with those of the earlier studies will be given later.

## Experimental

Essentially all of the reflections accessible to $\mathrm{Cu} K \alpha$ radiation were collected by Weissenberg photography on two crystals. The first crystal, with $c_{0}$ the rotation axis, was reduced to a cylinder 0.22 mm . in diameter, and zero-, first-, second- and third-layer-line data were recorded. The second crystal, with the zone axis [110] vertical, was a cylinder 0.35 mm . in diameter; the data of the layer lines zero to five were recorded. The small size of the crystals minimized absorption effects. Intensities were estimated by means of calibrated intensity strips and the multiple-film technique. The intensities of the strong reflections were estimated from powder photographs in order to eliminate extinction effects. Values for the lattice
constants were obtained by the method of least squares from measurements of 29 powder lines on photographs taken with $\mathrm{Cu} K \alpha$ radiation ( $\lambda=1.542 \AA$ ) in a powder camera of 5.73 cm . nominal radius. It is of interest to compare our values with those obtained in previous investigations; this comparison is made in Table 1.

Table 1. Lattice constants of urea

|  | $a_{0}(\AA)$ | $c_{0}(\AA)$ | $c / a$ |
| :--- | :---: | :---: | :---: |
| Mez (1902) | - | - | 0.8333 |
| Mark \& Weissenberg (1923) | $5 \cdot 62$ | 4.69 | 0.835 |
| Hendricks (1928) | $(5.71)$ | 4.76 | $(0.8333)$ |
| Wyckoff (1930) | 5.659 | 4.717 | 0.8335 |
| This work | 5.661 | 4.712 | 0.8325 |

## Refinement of parameters

The atomic parameters were refined by both Fourier and least-squares methods. Since the urea molecules lie in planes parallel to (110) and ( $1 \overline{1} 0$ ), only the section $\varrho\left(x, \frac{1}{2}+x, z\right)$ of the three-dimensional electron-density function was calculated. During the parameter refinement by the Fourier method twelve such sections were computed. Since the space group $P \overline{4} 2_{1} m$ lacks a center of symmetry it is necessary to assign a general phase angle, based on a calculated structure factor, to each observed value $\left|F_{h k l}\right|$. The terms in the Fourier series for the electron-density distribution thus depend not only on the observed structure-factor amplitudes, but on the values of the atomic parameters used, the atom form factors, and the contributions of the hydrogen atoms as well. The Fourier refinement included variation of all of these factors.

In the least-squares treatment of the data (Hughes, 1941) the observational equations used were of the form

$$
\begin{aligned}
& V w_{h k l} \cdot\left\{\ln \left(\left|F_{o}\right| /\left|F_{c}\right|\right)=-\Delta K-\Delta B_{1}\left(h^{2}+k^{2}\right)-\Delta B_{2} l^{2}+\right. \\
& \left.\quad+\frac{1}{\left|F_{c}\right|}\left(\frac{\partial F_{c}}{\partial z_{\mathrm{C}}} \Delta z_{\mathrm{C}}+\frac{\partial F_{c}}{\partial z_{\mathrm{C}}} \Delta z_{\mathrm{O}}+\frac{\partial F_{c}}{\partial z_{\mathrm{N}}} \Delta z_{\mathrm{N}}+\frac{\partial F_{c}}{\partial x_{\mathrm{N}}} \Delta x_{\mathrm{N}}\right)\right\}
\end{aligned}
$$

where $e^{K}$ is the scale factor and $B_{1}$ and $B_{2}$ are temperature factors. The approximate anisotropic temperature factors were determined from the peak shapes in the electron-density functions. Logarithms were used because $\ln \left|F_{c}\right|$ is linear in the temperature and scale factors, and because the logarithmic derivatives with respect to the atomic coordinates do not contain the temperature factor. Atom form factors of James \& Brindley were used in the structure factor calculations (Internationale Tabellen).

An IBM 604 calculating punch was used to calculate the coefficients in the observational equations, for the reduction to normal equations, and for structure-factor calculations. The solution of the seven normal equations was carried out in the following steps: (1) the four equations for the atomic positions were solved by neglecting the off-diagonal terms; (2) three iterations were made to complete the solution of these four
equations; (3) these solutions were put into the three scale and temperature-factor equations, which were then solved exactly for $K, B_{1}$, and $B_{2}$; (4) these results were put into the equations for the atomic positions to obtain a final solution for these parameters. Neglect of the off-diagonal terms in the solution of the equations for the atomic positions leads to an average change of $0.003 \AA$ and a maximum change of $0.004 \AA$. Neglect of the scale and temperature-factor terms in solving these equations produced an even smaller effect, the average change being $0 \cdot 0003 \AA$ and the maximum $0.0006 \AA$. We may thus conclude that the usual practice of solving for scale and temperature factors separately produces no significant error, and that neglect of the off-diagonal terms in the equations of atomic position leads to only a small decrease in accuracy, as has been previously pointed out (Shoemaker, Donohue, Schomaker \& Corey, 1950).

In the final electron-density calculation the phases were determined from structure factors for which an anisotropic temperature factor $\exp \left[-\sin ^{2} \theta / \lambda^{2}\left(3.9 \sin ^{2} \varphi_{h}+1.9 \cos ^{2} \varphi_{h}\right)\right]$ was applied to the form factors of carbon and oxygen, and the factor $\exp \left[-\sin ^{2} \theta / \lambda^{2}\left(3.9 \sin ^{2} \varphi_{h}+1.9 \cos ^{2} \varphi_{h}+\right.\right.$ $\left.\left.5 \cdot 7 \sin ^{2} \varphi_{h} \sin ^{2} \theta_{h}\right)\right]$ was applied to the form factor of nitrogen, where $\varphi_{h}$ and $\left(\theta_{h}+45^{\circ}\right)$ are the polar coordinates of the reciprocal-lattice vector, $h=2 \sin \theta / \lambda$. The numerical values in these factors were obtained from the least-squares treatment described above, and from measurement of the peak shapes in the electrondensity distributions. In addition, the contributions of the hydrogen atoms, the positions of which were suggested in the first electron-density sections, were included in the phase-angle determinations. The four hydrogen atoms were placed so that $\mathrm{N}-\mathrm{H}=\mathrm{I} \cdot 00 \AA$ and $\angle \mathrm{C}-\mathrm{N}-\mathrm{H}=120^{\circ}$. The final electron-density section, $\varrho\left(x, \frac{1}{2}+x, z\right)$, is shown in Fig. 2; an electron density section, $\varrho\left(x, y, z_{\mathrm{N}}\right)$, is shown in Fig. 3. Fourier sections $\varrho\left(x, \frac{1}{2}+x, z\right)$ were also calculated using calculated phases and amplitudes, in order to correct for series-termination errors (Booth, 1946). The anisotropic atom form factors were used in two such calculations, one of which included the scattering of the hydrogen atoms, the other did not. Atomic parameters at various stages of the work are presented in Table 2, together with the results of the previous work on urea.

The average between the second least-squares results and the final Fourier results were taken as the best values for the atomic parameters. The two methods gave answers which are in satisfactory agreement; the average difference is $0.005 \AA$, the


Fig. 2. Electron-density section $\varrho\left(x, \frac{1}{2}+x, z\right)$. Solid contour lines are drawn at $1,2,3, \ldots$ e. $\AA^{-3}$; the broken contour line is at $\frac{1}{2}$ e. $\AA^{-3}$. The small crosses indicate the positions assigned to the hydrogen atoms.


Fig. 3. Electron-density section $\varrho\left(x, y, z_{N}\right)$, showing the anisotropic shape of the nitrogen peak due to torsional motion of the molecule.
largest difference being that of $0.016 \AA$ in the case of $x_{\mathrm{N}}$. In three other crystals for which both threedimensional Fourier and least-squares were used, threonine (Shoemaker et al., 1950), hydroxyproline (Donohue \& Trueblood, 1952) and serine (Shoemaker, Barieau, Donohue \& Lu, 1952), the average differences

Table 2. Atomic parameters in urea

1. Mark \& Weissenberg
2. Hendricks
3. Wyckoff
4. Wyckoff \& Corey
5. From lst least squares
6. From 2nd least squares
7. $\varrho\left(x, \frac{1}{2}+x, z\right)$; phases without hydrogen contributions
8. $\varrho\left(x, \frac{1}{2}+x, z\right)$; phases with hydrogen contributions
9. Final values; average of 6 and 8

| $z_{\mathrm{C}}$ | $z_{\mathrm{O}}$ | $x_{\mathrm{N}}$ | $z_{\mathrm{N}}$ |
| :--- | :--- | :--- | :--- |
| - | -0.13 | - |  |
| 0.32 | 0.57 | 0.13 | 0.20 |
| 0.32 | 0.58 | 0.14 | 0.17 |
| 0.335 | 0.60 | 0.145 | 0.18 |
| 0.3319 | 0.5983 | 0.1450 | 0.1844 |
| 0.3305 | 0.5984 | 0.1443 | 0.1848 |
| 0.3322 | 0.5985 | 0.1421 | 0.1852 |
| 0.3310 | 0.5990 | 0.1415 | 0.1849 |
| 0.3308 | 0.5987 | 0.1429 | 0.1848 |

Table 3. Observed and calculated structure factors

| hkl | $\left\|F_{o}\right\|$ | $\left\|F_{c}\right\|$ | $\alpha\left({ }^{\circ}\right)$ | hkl | $\left\|F_{0}\right\|$ | $\left\|F_{c}\right\|$ | $\alpha\left({ }^{\circ}\right)$ | $h k l$ | $\left\|F_{0}\right\|$ | $\left\|F_{c}\right\|$ | $\alpha\left({ }^{\circ}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 200 | 10.88 | 12.38 | 0 | 521 | $1 \cdot 70$ | 1.01 | 14-1 | 113 | 3-19 | $2 \cdot 58$ | 221.2 |
| 400 | $2 \cdot 20$ | $0 \cdot 69$ | 0 | 621 | $1 \cdot 40$ | 1.75 | 147-1 | 213 | $5 \cdot 82$ | 5.94 | $137 \cdot 9$ |
| 600 | $3 \cdot 42$ | $3 \cdot 62$ | 0 | 331 | $2 \cdot 44$ | 2.53 | $24 \cdot 6$ | 313 | $5 \cdot 59$ | $5 \cdot 61$ | 184.1 |
| 110 | $30 \cdot 17$ | 28.17 | 180 | 431 | $2 \cdot 88$ | $2 \cdot 11$ | $264 \cdot 0$ | 413 | 1.93 | 1.82 | $57 \cdot 8$ |
| 210 | 11.80 | 11.51 | 0 | 531 | 1.70 | 1.75 | $335 \cdot 4$ | 513 | 1.75 | $2 \cdot 02$ | 166.0 |
| 310 | $4 \cdot 44$ | $4 \cdot 30$ | 180 | 631 | $<0.43$ | 0.35 | $117 \cdot 2$ | 613 | $1 \cdot 16$ | 1.31 | 57.7 |
| 410 | $1 \cdot 15$ | $1 \cdot 68$ | 180 | 441 | $1 \cdot 50$ | 1-29 | $192 \cdot 3$ | 223 | $4 \cdot 62$ | 3.99 | $28 \cdot 1$ |
| 510 | $3 \cdot 85$ | $3 \cdot 44$ | 180 | 541 | $0 \cdot 64$ | 0.43 | 116.2 | 323 | $3 \cdot 10$ | $3 \cdot 41$ | 296.9 |
| 610 | $0 \cdot 88$ | 1-11 | 180 | 551 | $1 \cdot 61$ | 1.09 | $32 \cdot 4$ | 423 | $1 \cdot 45$ | $1 \cdot 63$ | 345.6 |
| 710 | 1.97 | 2.09 | 180 | 002 | $9 \cdot 70$ | 9.73 | 180 | 523 | $2 \cdot 03$ | 1.97 | $222 \cdot 2$ |
| 220 | $10 \cdot 34$ | 11.69 | 0 | 102 | $9 \cdot 89$ | $9 \cdot 80$ | 90 | 333 | $0 \cdot 64$ | $0 \cdot 40$ | $240 \cdot 0$ |
| 320 | 3-13 | 3.17 | 180 | 202 | $2 \cdot 27$ | $2 \cdot 34$ | 0 | 433 | $2 \cdot 24$ | $2 \cdot 14$ | $80 \cdot 9$ |
| 420 | $6 \cdot 14$ | $5 \cdot 94$ | 0 | 302 | $2 \cdot 89$ | $2 \cdot 37$ | 270 | 533 | 0.80 | 1.02 | 168.5 |
| 520 | $2 \cdot 18$ | $2 \cdot 33$ | 0 | 402 | 1.80 | 2.57 | 0 | 443 | $<0.55$ | 0.33 | $14 \cdot 1$ |
| 620 | $2 \cdot 50$ | $1 \cdot 99$ | 0 | 502 | $0 \cdot 68$ | 0.08 | 90 | 004 | $4 \cdot 17$ | $4 \cdot 59$ | 180 |
| 330 | 8.85 | 9.71 | 180 | 602 | < 0.69 | $0 \cdot 82$ | 180 | 104 | $1 \cdot 30$ | $0 \cdot 64$ | 90 |
| 430 | $1 \cdot 62$ | 0.58 | 180 | 112 | $5 \cdot 76$ | $5 \cdot 19$ | $61 \cdot 1$ | 204 | $3 \cdot 62$ | $3 \cdot 62$ | 180 |
| 530 | 3.57 | 3.17 | 180 | 212 | $4 \cdot 73$ | $4 \cdot 70$ | 206.4 | 304 | 6.34 | 6.64 | 90 |
| 630 | $<0.48$ | 0.37 | 180 | 312 | $3 \cdot 11$ | $2 \cdot 93$ | $142 \cdot 0$ | 404 | $1 \cdot 82$ | $1 \cdot 91$ | 180 |
| 440 | 4.65 | $4 \cdot 61$ | 0 | 412 | $1 \cdot 03$ | 0.99 | $33 \cdot 1$ | 504 | $2 \cdot 20$ | $2 \cdot 13$ | 90 |
| 540 | < 0.59 | $0 \cdot 47$ | 180 | 512 | 1.09 | 1-34 | 266.5 | 114 | 4.98 | $5 \cdot 11$ | $320 \cdot 6$ |
| 550 | $0 \cdot 80$ | 1.53 | 180 | 612 | $0 \cdot 69$ | $0 \cdot 88$ | 313.9 | 214 | $4 \cdot 69$ | $4 \cdot 34$ | 268.5 |
| 001 | 4.02 | $4 \cdot 33$ | 180 | 222 | $5 \cdot 30$ | $4 \cdot 85$ | 262.8 | 314 | 2.95 | $2 \cdot 89$ | 334.6 |
| 101 | 10.66 | $10 \cdot 19$ | 90 | 322 | 1.92 | 2.56 | 58.5 | 414 | 3.24 | 3.72 | $270 \cdot 6$ |
| 201 | 10.88 | 10.87 | 180 | 422 | $1 \cdot 21$ | $1 \cdot 09$ | 119.7 | 514 | $1 \cdot 84$ | $1 \cdot 88$ | $42 \cdot 3$ |
| 301 | $7 \cdot 10$ | $7 \cdot 65$ | 270 | 522 | $1 \cdot 26$ | I-34 | 158.9 | 224 | $4 \cdot 44$ | $4 \cdot 72$ | 126.7 |
| 401 | 6.24 | 6.31 | 180 | 622 | $0 \cdot 64$ | 0.65 | 96.2 | 324 | $2 \cdot 63$ | 2.06 | $89 \cdot 1$ |
| 501 | $<0.60$ | $0 \cdot 66$ | 270 | 332 | 1.64 | 1.98 | 12.8 | 424 | 2.04 | $2 \cdot 01$ | $205 \cdot 2$ |
| 601 | 1.29 | 1-17 | 180 | 432 | $2 \cdot 16$ | 1.96 | 278.5 | 524 | 1.05 | $1 \cdot 36$ | $92 \cdot 1$ |
| 701 | 0.97 | $1 \cdot 10$ | 90 | 532 | 0.91 | 0.53 | 308.5 | 334 | $2 \cdot 16$ | 2.01 | 348.6 |
| 111 | 11.74 | 14.25 | 53.2 | 632 | $<0.43$ | 0.28 | $44 \cdot 4$ | 005 | $5 \cdot 14$ | $5 \cdot 17$ | 0 |
| 211 | $5 \cdot 83$ | $5 \cdot 18$ | $25 \cdot 2$ | 442 | 1.03 | 1.07 | 191.2 | 105 | 1-46 | $2 \cdot 76$ | 270 |
| 311 | 8.77 | $8 \cdot 64$ | 16.7 | 542 | $<0.48$ | $0 \cdot 43$ | $52 \cdot 8$ | 205 | 0.91 | 0.34 | 0 |
| 411 | 2.01 | $2 \cdot 72$ | $104 \cdot 9$ | 003 | $2 \cdot 45$ | $3 \cdot 21$ | 180 | 305 | $<0.46$ | 0.07 | 270 |
| 511 | 2.90 | $3 \cdot 40$ | $324 \cdot 6$ | 103 | $7 \cdot 45$ | 7.58 | 270 | 115 | 2.95 | $2 \cdot 83$ | $202 \cdot 2$ |
| 611 | $1 \cdot 23$ | $0 \cdot 86$ | $239 \cdot 2$ | 203 | $5 \cdot 25$ | $5 \cdot 40$ | 0 | 215 | $2 \cdot 34$ | $2 \cdot 60$ | 28.7 |
| 711 | 0.75 | $0 \cdot 72$ | 358.4 | 303 | $2 \cdot 16$ | 1.72 | 270 | 315 | $0 \cdot 68$ | $0 \cdot 65$ | $317 \cdot 6$ |
| 221 | 10.88 | 10.56 | 227.5 | 403 | 4.92 | $5 \cdot 60$ | 0 | 225 | $1 \cdot 46$ | 1.59 | $52 \cdot 8$ |
| 321 | 1.38 | $1 \cdot 49$ | $152 \cdot 4$ | 503 | 1.30 | 1.31 | 270 | 325 | $1 \cdot 68$ | $1 \cdot 56$ | $236 \cdot 1$ |
| 421 | $3 \cdot 20$ | $3 \cdot 37$ | $155 \cdot 7$ | 603 | 0.79 | $0 \cdot 34$ | 0 | 006 | $2 \cdot 27$ | $2 \cdot 43$ | 0 |

were $0.005,0.008$ and $0.006 \AA$, and the maximum differences were $0.016,0.023$ and $0.013 \AA$, respectively.

Application of the theory of errors to the leastsquares results gives standard errors of $0.015,0.009$, 0.012 and $0.008 \AA$ in $z_{\mathrm{C}}, z_{0}, x_{\mathrm{N}}$ and $z_{\mathrm{N}}$, respectively; the average is $0.011 \AA$. If we assume that the $0.004 \AA$ r.m.s. deviation from the average of parameters determined by least-squares and Fourier methods represents an approximate standard error due to choice of the method of interpretation (Shoemaker et al., 1950), the total average standard error in a single parameter is $0.012 \AA$, which corresponds to a probable error of $0.008 \AA$. Probable errors in the molecular parameters are thus $0.011 \AA$ for $\mathrm{C}-0,0.009 \AA$ for $\mathrm{C}-\mathrm{N}, 0.45^{\circ}$ for $\angle \mathrm{O}-\mathrm{C}-\mathrm{N}$ and $0.9^{\circ}$ for $\angle \mathrm{N}-\mathrm{C}-\mathrm{N}$.

Table 3 gives the observed and calculated values of $\left|F_{\text {hkl }}\right|$. The calculated values include the contributions of the hydrogen atoms. The average discrepancy, $R$, is $9.9 \%$. If the hydrogen scattering is neglected $R$ is $11.0 \%$. If the torsional motion of the nitrogen atom is also neglected $R$ is $12 \cdot 4 \%$.

## Discussion

Interatomic distances and interbond angles in the urea crystal are listed in Table 4.

Table 4. Bond distances and angles in urea

| $\mathrm{C}-\mathrm{O}$ | $1.262 \AA$ |
| :--- | :--- |
| $\mathrm{C}-\mathrm{N}$ | 1.35 A |
| $\angle \mathrm{C}-\mathrm{N}$ | $118.0^{\circ}$ |
| $\angle \mathrm{N}-\mathrm{C}-\mathrm{O}$ | $1210^{\circ}$ |
| $\mathrm{N} \cdots \mathrm{O}$ | $2.989 \AA$ |
| $\angle \mathrm{C}-\mathrm{N} \cdots \mathrm{O}$ | $98.5^{\circ} \AA$ |
| $\mathrm{N} \cdots \mathrm{O}^{\prime}$ | $3.035 \AA$ |
| $\angle \mathrm{C}-\mathrm{N} \cdots \mathrm{O}^{\prime}$ | $129.3^{\circ}$ |

The hydrogen bond distances, $\mathrm{N} \cdots \mathrm{O}=2.99$ and $3.04 \AA$, correspond to rather weak bonds, since both are considerably longer than the $\mathrm{N} \cdots \mathrm{O}$ distances of 2.7 to $2.9 \AA$ found in the amino acids. This relative lengthening is without doubt due to (1) the acceptance of a large number (four) of hydrogen bonds by the oxygen atom in urea, and (2) the smaller formal charge on the nitrogen atom in urea; in the amino acids this atom, because of the Zwitterion structure, has a charge of +1 , while in urea it has a smaller
positive charge, depending on the relative contributions of the following resonance structures:

(A)

(B)

(C)

In order to estimate the relative contributions of these structures we shall use the familiar equation of Pauling (1940) which relates bond length with double-bond character. It is then necessary to have reliable values for the lengths of carbon-oxygen and carbon-nitrogen single and double bonds. Satisfactory agreement with observed results is obtained if the revised values $1.20 \AA$ for $\mathrm{C}=\mathrm{O}$ and $1.24 \AA$ for $\mathrm{C}=\mathrm{N}$ are used, combined with the values $1.42 \AA$ for $\mathrm{C}-\mathrm{O}$ and $1.48 \AA$ for C-N adopted by Schomaker \& Stevenson (1941). The values for $C=O$ and $C=N$ are 0.015 and $0.025 \AA$ shorter than those given by Pauling (1940). In carboxylic acids there is resonance between two forms, -C lative contributions of the two forms depending on
the situations of the two oxygen atoms, i.e. the number and strengths of the hydrogen bonds they are accepting, or whether one of them is covalently bonded to a hydrogen atom, as in the case of the free acids. The total double-bond character to be distributed between the two carbon oxygen bonds is 1.00 , so that if a percentage of double bond character is assigned to one bond the lengths of both bonds can be predicted. Comparison of observed with calculated bond lengths is made in Table 5.

The data in Table 5 show that excellent agreement between calculated and observed bond distances is obtained by the use of Pauling's equation and the distances $\mathrm{C}-\mathrm{O}=1.42 \AA$ and $\mathrm{C}=\mathrm{O}=1.20 \AA$.

Calculated and observed values for carbon-oxygen and carbon-nitrogen bonds in peptides and related compounds are presented in Table 6. The agreement is again satisfactory. The results indicate that in urea structures $A, B$ and $C$ contribute $40 \%, 30 \%$ and $30 \%$, respectively. We therefore expect the entire urea molecule, including the hydrogen atoms, to be planar, a conclusion verified by the electron-density sections, the disposition of the hydrogen bonds, and the results of spectroscopic investigations of urea crystals with polarized infra-red (Keller, 1948; Waldron \& Badger, 1950). Moreover, unsymmetrical structures such as

Table 5. Bond lengths in carboxylic acids

| Compound | Double-bond character | $\mathrm{C}-\mathrm{O}$, obs. <br> ( $\AA$ ) | $\mathrm{C}-\mathrm{O}$, calc. ( $\AA$ ) | Reference |
| :---: | :---: | :---: | :---: | :---: |
| Adipic acid | $\begin{aligned} & 0.30 \\ & 0.70 \end{aligned}$ | $\begin{aligned} & 1 \cdot 29 \\ & 1.23 \end{aligned}$ | $\begin{aligned} & 1.30 \\ & 1.23 \end{aligned}$ | Morrison \& Robertson (1949b) |
| Glutaric acid | $\begin{aligned} & 0.30 \\ & 0.70 \end{aligned}$ | $\begin{aligned} & 1 \cdot 30 \\ & 1 \cdot 23 \end{aligned}$ | $\begin{aligned} & 1.30 \\ & 1.23 \end{aligned}$ | Morrison \& Robertson (1949d) |
| Sebacic acid | $\begin{aligned} & 0.40 \\ & 0.60 \end{aligned}$ | $\begin{aligned} & 1.27 \\ & 1.24 \end{aligned}$ | $\begin{aligned} & 1.27 \\ & 1.24 \end{aligned}$ | Morrison \& Robertson (1949c) |
| Succinic acid | $\begin{aligned} & 0.30 \\ & 0.70 \end{aligned}$ | $\begin{aligned} & 1.30 \\ & 1 \cdot 25 \end{aligned}$ | $\begin{array}{r} 1.30 \\ 1.23 \end{array}$ | Morrison \& Robertson (1949a) |
| Acetylglycine | $\begin{aligned} & 0.25 \\ & 0.75 \end{aligned}$ | $\begin{aligned} & 1.31 \\ & 1.19 \end{aligned}$ | $\begin{array}{r} 1.31 \\ 1.22 \end{array}$ | Carpenter \& Donohue (1950) |
| Alanine | $\begin{aligned} & 0.35 \\ & 0.65 \end{aligned}$ | $\begin{aligned} & 1 \cdot 27 \\ & 1 \cdot 21 \end{aligned}$ | $\begin{aligned} & 1 \cdot 28 \\ & 1 \cdot 23 \end{aligned}$ | Donohue (1949) |
| Glycylglycine | $\begin{aligned} & 0.35 \\ & 0.65 \end{aligned}$ | $\begin{aligned} & 1 \cdot 27 \\ & 1 \cdot 21 \end{aligned}$ | $\begin{aligned} & 1.28 \\ & 1.23 \end{aligned}$ | Hughes \& Moore (1949) |
| Hydroxyproline | $\begin{aligned} & 0.40 \\ & 0.60 \end{aligned}$ | $\begin{array}{r} 1.27 \\ 1.25 \end{array}$ | $\begin{aligned} & 1.27 \\ & 1.24 \end{aligned}$ | Donohue \& Trueblood (1952) |
| Serine | $\begin{aligned} & 0.50 \\ & 0.50 \end{aligned}$ | $\begin{aligned} & 1.27 \\ & 1.26 \end{aligned}$ | $\begin{aligned} & 1.26 \\ & 1.26 \end{aligned}$ | Shoemaker, Barieau, Donohue \& Lu (1952) |
| Threonine | $\begin{aligned} & 0.50 \\ & 0.50 \end{aligned}$ | $\begin{array}{r} 1.25 \\ 1.24 \end{array}$ | $\begin{aligned} & 1.26 \\ & 1.26 \end{aligned}$ | Shoemaker, Donohue, Schomaker \& Corey (1950) |
| Formic Acid monomer | $\begin{aligned} & 0.10 \\ & 0.90 \end{aligned}$ | $\begin{aligned} & 1 \cdot 37 \\ & 1 \cdot 21 \end{aligned}$ | $\begin{aligned} & 1 \cdot 37 \\ & 1 \cdot 21 \end{aligned}$ | Schomaker \& O'Gorman (1947) |
| Carbonate ion | 0.33 | 1.31 | 1.29 | Elliott (1937) |

Table 6. Bond distances in peptides and related compounds

| Compound | Bond | Double-bond character | Distance, obs. ( $\AA$ ) | Distance, calc. (A) | Reference |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Acetamide | $\mathrm{C}-\mathrm{O}$ | 0.70 | 1-28 | 1.23 | Senti \& Harker (1940) |
|  | $\mathbf{C - N}$ | 0.30 | $1 \cdot 38$ | $1 \cdot 35$ |  |
| Acetylglycine | $\mathrm{C}-\mathrm{O}$ | $0 \cdot 60$ | 1. 24 | 1.24 | Carpenter \& Donohue (1950) |
|  | $\mathbf{C - N}$ | $0 \cdot 40$ | $1 \cdot 32$ | 1-32 |  |
| Diketopiperazine | $\mathrm{C}-\mathrm{O}$ | 0.60 | $1 \cdot 25$ | $1 \cdot 24$ | Corey (1938) |
|  | $\mathrm{C}-\mathrm{N}$ | $0 \cdot 40$ | 1.33 | 1.32 |  |
| Glycylglycine | $\mathrm{C}-\mathrm{O}$ | $0 \cdot 60$ | $1 \cdot 23$ | 1.24 | Hughes \& Moore (1949) |
|  | C-N | $0 \cdot 40$ | $1 \cdot 29$ | $1 \cdot 32$ |  |
| Polypeptide chain | C-O | 0.65 | 1.23 | $1 \cdot 23$ | Corey \& Donohue (1950) |
|  | C-N | 0.35 | $1 \cdot 32$ | $1 \cdot 33$ |  |
| Urea | C-O | 0.40 | 1-26 | 1.27 | This work |
|  | $\mathrm{C}-\mathrm{N}$ | 0.30 | 1.34 | $1 \cdot 35$ |  |
| $\text { Urea. } \mathrm{H}_{2} \mathrm{O}_{2}$ | $\mathrm{C}-\mathrm{O}$ | 0.50 | 1.24 | 1.25 | Lu, Hughes \& Giguere (1941) |
|  | $\mathrm{C}-\mathrm{N}$ | $0 \cdot 25$ | 1-34 | I-36 |  |

$-\mathrm{O}-\mathrm{C} \mathbb{N}^{\mathrm{N}+\mathrm{H}_{3}}{ }^{\mathrm{NH}}$, which Clow (1937) proposed on the basis of magnetic susceptibility measurements, are not in agreement with the above results.

Also of interest is the result that the interatomic distances in the polypeptide chain structure formulated by Corey \& Donohue (1950) correspond to contributions of $65 \%$ and $35 \%$ for the structures

and


The contribution of the second of these structures leads to the planarity of the peptide group and its associated atoms, a situation which has already been emphasized and incorporated into detailed considerations of the configuration of polypeptide chains in protein molecules (Pauling \& Corey, 1950; Pauling, Corey \& Branson, 1951).

This work was aided by a grant from the National Foundation for Infantile Paralysis.

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[^0]:    * Contribution No. 1650 from the Gates and Crellin Laboratories.
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