Section 10.1

Definition: A **plane** curve is a set C of ordered pairs $(f(t), g(t))$, where f and g are continuous functions on an interval I.

Definition: The **graph** of C in the definition consists of all points $P(t) = (f(t), g(t))$ in an xy-plane, for t in I.

Definition: Let C be the curve consisting of all ordered pairs $(f(t), g(t))$, where f and g are continuous on an interval I. The equations

$$x = f(t),\ y = g(t),$$

for t in I, are **parametric equations** for C with **parameter** t.

The curve C in this definition is referred to as a **parametrized curve**, and the parametric equations are a **parametrization** for C. We often use the notation

$$x = f(t), \quad y = g(t); \quad t \text{ in } I$$

to indicate the domain I of f and g.

Exercise 1. Sketch the graph of the curve C that has the parametrization

$$x = 2t,\ y = t^2 - 1; \quad -1 \le t \le 2.$$

Class Exercise 1. Sketch the curve by using the parametric equations to plot the points. Indicate with an arrow the direction in which the curve is traced as t increases.
(a) $x = t^2$, $y = t^3 - 4t$, $-3 \le t \le 3$
(b) $x = e^{-t} + t$, $y = e^t - t$, $-2 \le t \le 2$

Exercise 2. A point moves in a plane such that its position $P(x, y)$ at time t is given by

$$x = a \cos t,\ y = a \sin t;\ -\infty < t < \infty,$$

where $a > 0$. Describe the motion of the point.

Class Exercise 2. (a) Sketch the curve by using the parametric equations to plot the points. Indicate with an arrow the direction in which the curve is traced as t increases.

(b) Eliminate the parameter to find a Cartesian equation of the curve.
(i) $x = 1 - 2t$, $y = \frac{1}{2}t - 1$, $-2 \le t \le 4$
(ii) $x = t - 1$, $y = t^3 + 1$, $-2 \le t \le 2$
(iii) $x = t^2$, $y = t^3$, $-2 \le t \le 2$

Exercise 3. Sketch the graph of the curve C that has the parametrization

$$x = -2 + t^2,\ y = 1 + 2t^2;\ -\infty < t < \infty$$

and indicate the orientation.

Class Exercise 3. (a) Eliminate the parameter to find a Cartesian equation of the curve.
(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.
(i) $x = \frac{1}{2} \cos\theta$, $y = 2 \sin\theta$, $0 \le \theta \le \pi$
(ii) $x = e^t - 1$, $y = e^{2t}$
(iii) $x = \sqrt{t+1}$, $y = \sqrt{t-1}$
(iv) $x = \tan^2\theta$, $y = \sec\theta$, $-\pi/2 < \theta < \pi/2$

Homework: 3-21 ODD, 25-33 ODD