Section 10.3

In a rectangular coordinate system, the ordered pair (a, b) denotes the point whose directed distances from the $x-$ and $y-$axes are b and a, respectively. Another method for representing points is to use *polar coordinates*.

To define polar coordinates, we first fix an origin O (call the **pole**) and an **initial ray** from O. Usually the positive x-axis is chosen as the initial ray. Then the point P can be located by assigning to it a **polar coordinate pair** (r, θ) in which r gives the directed distance from O to P and θ gives the directed angle from the initial ray to ray OP. So we label the point P as $P(r, \theta)$.

Exercise 1. Find all the polar coordinates of the point $P(2, \pi/6)$.

Exercise 2. Plot the points with the given polar coordinates.
(a) $(-3, 5\pi/6)$ (b) $(5, \tan^{-1}(4/3))$ (c) $(-1, 7\pi)$ (d) $(2\sqrt{3}, 2\pi/3)$

Class Exercise 1. Plot the point whose polar coordinates are given.
(a) $(-\sqrt{2}, 5\pi/4)$ (b) $(1, 5\pi/2)$ (c) $(2, -7\pi/6)$

Exercise 3. Sketch the graph of the following polar equations:
(a) $r = 4 \sin\theta$ (b) $r = 2 + 2\cos\theta$ (c) $r = 2 + 4\cos\theta$ (d) $r = a \sin\theta$

Class Exercise 2. Graph the polar curve. (a) $r = 1 + \cos\theta$
(b) $r = 2 - 2\cos\theta$ (c) $r^2 =$ -sin 2θ (d) $r = 1$ - $\sin\theta$
(e) $r = 1$ - $2\sin 3\theta$ (f) $r = \sin(\theta/2)$ (g) $r = \theta$
(h) $r = 1 + \sin\theta$ (i) $r = 2\cos 3\theta$ (j) $r = 1 + 2\sin\theta$

Relationship between rectangular and polar coordinates: The rectangular coordinates (x, y) and polar coordinates (r, θ) of a point P are related as follows:
(i) $x = r\cos\theta$, $y = r\sin\theta$

(ii) $r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$ if $x \neq 0$

Exercise 4. Find the Cartesian coordinates for the following points that are given in polar coordinates:
(a) $(\sqrt{2}, \pi/4)$ (b) $(1, 0)$ (c) $(0, \pi/2)$ (d) $(-\sqrt{2}, \pi/4)$

Class Exercise 3. Find the Cartesian coordinates for the following points that are given in polar coordinates:
(a) $(-3, 5\pi/6)$ (b) $(5, \tan^{-1}(4/3))$ (c) $(-1, 7\pi)$ (d) $(2\sqrt{3}, 2\pi/3)$

Exercise 5. Find an equation in x and y that has the same graph as the polar equation $r = a\sin\theta$, with $a \neq 0$. Sketch the graph.

Exercise 6. Replace the following polar equations by equivalent Cartesian equations and identify their graphs.
(a) $r\cos\theta =$ -4 (b) $r^2 = 4r\cos\theta$ (c) $r = \frac{4}{2\cos\theta - \sin\theta}$

Class Exercise 4. Replace the polar equation by an equivalent Cartesian equation. Then identify or describe the graph.
(a) $r\sin\theta = 0$ (b) $r\cos\theta = 0$ (c) $r = 4\csc\theta$
(d) $r =$ -3 $\sec\theta$ (e) $r\cos\theta + r\sin\theta = 1$
(f) $r^2 = 1$ (g) $r^2 = 4r\sin\theta$ (h) $r = \frac{5}{\sin\theta - 2\cos\theta}$
(i) $r^2\sin 2\theta = 2$
(j) $r = \cot\theta\csc\theta$

Homework: 1-25 (every 4th), 33-49 (every 4th)