Section 11.11

Theorem: Let f have $n+1$ derivatives throughout an interval containing c. If x is any number in the interval and $x \neq c$, then the error in approximating $f(x)$ by the nth degree Taylor polynomial of f at c,

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n,$$

is less than $|R_n(x)|$, where

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$$

and z is the number between c and x.

Exercise 1. Let $f(x) = \ln x$.
(a) Find $P_3(x)$ and $R_3(x)$ at $c = 1$.
(b) Approximate ln 1.1 to four decimal places by means of $P_3(1.1)$, and use $R_3(1.1)$ to estimate the error in this approximation.

Class Exercise 1. (a) Approximate f by a Taylor polynomial with degree n at the number a.
(b) Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the given interval.
(i) $f(x) = x^{-2}$, $a = 1$, $n = 2$, $0.9 \leq x \leq 1.1$
(ii) $f(x) = \sin x$, $a = \pi/6$, $n = 4$, $0 \leq x \leq \pi/3$
(iii) $f(x) = \ln(1+2x)$, $a = 1$, $n = 3$, $0.5 \leq x \leq 1.5$
(iv) $f(x) = x \ln x$, $a = 1$, $n = 3$, $0.5 \leq x \leq 1.5$
(v) $f(x) = \sinh 2x$, $a = 0$, $n = 5$, $-1 \leq x \leq 1$

Exercise 2. Use a Taylor polynomial to approximate cos 61°, and estimate the accuracy of the approximation.

Class Exercise 2. Use a Taylor polynomial to approximate sin 38°, and estimate the accuracy of the approximation.

Class Exercise 3. Suppose you know that

$$f^{(n)}(4) = \frac{(-1)^n \cdot n!}{3^n(n+1)}$$

and the Taylor series of f centered at 4 converges to $f(x)$ for all x in the interval of convergence. Show that the fifth-degree Taylor polynomial approximates $f(5)$ with error less than 0.0002.

Homework: 3-31 (every 4th)