Section 11.4

Exercise From the Past: Does the integral $\int_1^\infty e^{-x^2}\,dx$ converge?

Basic Comparison Test: Let $\sum a_n$ and $\sum b_n$ be positive term series.
(i) If $\sum b_n$ converges and $a_n \le b_n$ for every positive integer n, then $\sum a_n$ converges.
(ii) If $\sum b_n$ diverges and $a_n \ge b_n$ for every positive integer n, then $\sum a_n$ diverges.

Exercise 1. Determine whether the series converges or diverges:
(a) $\sum_{n=1}^{\infty} \frac{1}{2+5^n}$
(b) $\sum_{n=2}^{\infty} \frac{3}{\sqrt{n}-1}$

Class Exercise 1. Determine whether the series converges or diverges.
(a) $\sum_{n=2}^{\infty} \frac{n^3}{n^4-1}$
(b) $\sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n}}$
(c) $\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$
(d) $\sum_{k=1}^{\infty} \frac{k\sin^2 k}{1+k^3}$

Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be positive term series. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0,$$

then either both series converge or both diverge.

Exercise 2. Determine whether the series converges or diverges:
(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+1}}$
(b) $\sum_{n=1}^{\infty} \frac{3n^2+5n}{2^n(n^2+1)}$

Exercise 3. Let $a_n = \frac{8n+\sqrt{n}}{5+n^2+n^{7/2}}$. Determine whether $\sum a_n$ converges or diverges.

Class Exercise 2. Determine whether the series converges or diverges.
(a) $\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$
(b) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$
(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{3n^4+1}}$
(d) $\sum_{n=1}^{\infty} \frac{1}{2n+3}$
(e) $\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$
(f) $\sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3}$
(g) $\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$
(h) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2-1}}$
(i) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$
(j) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
(k) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$

Homework: 7-27 (every 4th), 33, 37, 43