

Section 11.6

Ratio Test: Let $\sum a_n$ be a series of nonzero terms, and suppose

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L.$$

- (i) If $L < 1$, the series is absolutely convergent.
- (ii) If $L > 1$ or $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$, the series is divergent.
- (iii) If $L = 1$, apply a different test; the series may be absolutely convergent, conditionally convergent, or divergent.

Exercise 5. Determine the convergence of the series: (a) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ (b) $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$.

Exercise 6. Determine the convergence of $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

Class Exercise 1. Investigate the convergence of the following series.

(a) $\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n+1)!}$ (b) $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$ (c) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$ (d) $\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$

The Root Test

- (i) If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
- (ii) If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L > 1$ or $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- (iii) If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = 1$, the Root Test is inconclusive.

Exercise 7. Determine the convergence or divergence of $\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n^n}$?

Class Exercise 2. Which of the following series converge, and which diverge?

(a) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ (b) $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ (c) $\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$

Class Exercise 3. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$ (b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$ (c) $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$ (d) $\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$ (e) $\sum_{n=1}^{\infty} \frac{3-\cos n}{n^{2/3}-2}$
 (f) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ (g) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$ (h) $\sum_{n=2}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$ (i) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ (j) $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$

Homework: 3-19 (every 4th), 21-33 (every 4th)