Section 12.1

Definition: In order to represent points in space, we first choose a fixed point O (the origin) and three directed lines through O that are perpendicular to each other, called the **coordinate axes** and labeled the x-axis, y-axis, and z-axis.

Definition: The three coordinate axes determine the three **coordinate planes**.

Definition: Now if P is any point in space, let a be the (directed) distance from the yz-plane to P, let b be the distance from the xz-plane to P, and let c be the distance from the xy-plane to P. We represent the point P by the ordered triple (a, b, c) of real numbers and we call a, b, and c the coordinates of P; a is the x-coordinate, b is the y-coordinate, and c is the z-coordinate. Thus, to locate the point (a, b, c), we can start at the origin O and move a units along the x-axis, then b units parallel to the y-axis, and the c units parallel to the z-axis.

Definition: The point $P(a, b, c)$ determines a rectangular box. If we drop a perpendicular from P to the xy-plane, we get a point Q with coordinates $(a, b, 0)$ called the **projection** of P onto the xy-plane. Similarly, $R(0, b, c)$ and $S(a, 0, c)$ are the projections of P onto the yz-plane and xz-plane, respectively.

Exercise 1. Sketch the points $(0, 1, 5)$, $(3, 1, 6)$, and $(-1, 1, 4)$ on a single set of coordinate axes.

Class Exercise 1. Sketch the points $(0, 5, 2)$, $(4, 0, -1)$, $(2, 4, 6)$, and $(1, -1, 2)$ on a single set of coordinate axes. (#2)

Exercise 2. What does the equation $x = 4$ represent in $\mathbb{R}^2$? What does it represent in $\mathbb{R}^3$? Illustrate with sketches.

Exercise 3. What does the equation $y = 3$ represent in $\mathbb{R}^3$? What does $z = 5$ represent? What does the pair of equation $y = 3$, $z = 5$ represent? In other words, describe the set of points (x, y, z) such that $y = 3$ and $z = 5$. Illustrate with a sketch. (#6)

Distance Formula: The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Exercise 4. Find the distance between $P_1(2, 1, 5)$ and $P_2(-2, 3, 0)$. (Hass Example 3)

Class Exercise 2. Suppose we have points $P(2, -1, 0)$, $Q(4, 1, 1)$, and $R(4, -5, 4)$. Find the lengths of the sides of the triangle PQR. Is it a right triangle? It is an isosceles triangle? (#8)

Equation of a Sphere: An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2.$$

Exercise 5. Find the center and radius of the sphere: $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$.
(Hass Sec 12.1 Example 4)

Class Exercise 3. Find an equation of the sphere with center $(2, -6, 4)$ and radius 5. Describe its intersection with each of the coordinate planes. (#12)

Class Exercise 4. Find an equation of the sphere that passes through the origin and whose center is (1,2,3). (#14)

Class Exercise 5. Show that the equation represents a sphere, and find its center and radius:
(a) $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$.
(b) $3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z$. (#16, 18)

Class Exercise 6. Describe in words the region of $\mathbb{R}^3$ represented by the equations or inequalities. (#24-34 even)
(a) $y = 2$ (b) $x \geq -3$ (c) $z^2 = 1$
(d) $y^2 + z^2 = 16$ (e) $x = z$ (f) $x^2 + y^2 + z^2 > 2z$

Homework: 1, 5, 9, 11-31 (every 4th), 37, 41