Section 12.2

Definition: The term **vector** is a quantity that has both magnitude and direction.

Definition: For instance, suppose a particle moves along a line segment from point A to point B. The corresponding **displacement vector** $\overrightarrow{v}$ has **initial point** A (the tail) and **terminal point** B (the tip) and we indicate this by writing $\overrightarrow{v} = \overrightarrow{AB}$.

Definition: Suppose that $\overrightarrow{u}$ and $\overrightarrow{v}$ have the same length and direction. We say that $\overrightarrow{u}$ and v are **equivalent** and we write $\overrightarrow{u} = \overrightarrow{v}$.

Definition: The **zero vector**, denoted by $\overrightarrow{0}$, has length 0. It is the only vector with no specific direction.

Definition: If $\overrightarrow{u}$ and $\overrightarrow{v}$ are vectors positioned so the initial point of $\overrightarrow{v}$ is at the terminal point of $\overrightarrow{u}$, the **sum** $\overrightarrow{u} + \overrightarrow{v}$ is the vector from the initial point of $\overrightarrow{u}$ to the terminal point of $\overrightarrow{v}$.

Definition: If c is a scalar and $\overrightarrow{v}$ is a vector, then the **scalar multiple** $c\overrightarrow{v}$ is the vector whose length is $|\ c\ |$ times the length of $\overrightarrow{v}$ and whose direction is the same as $\overrightarrow{v}$ if $c > 0$ and is opposite to $\overrightarrow{v}$ if $c < 0$. If $c = 0$ and $\overrightarrow{v} = \overrightarrow{0}$, then $c\overrightarrow{v} = \overrightarrow{0}$.

Definition: Notice that two nonzero vectors are **parallel** if they are scalar multiples of one another.

Definition: The vector $-\overrightarrow{v} = (-1)\overrightarrow{v}$ has the same length as $\overrightarrow{v}$ but points in the opposite direction. We call it the **negative** of $\overrightarrow{v}$.

Definition: By the difference $\overrightarrow{u} - \overrightarrow{v}$ of two vectors we mean

$$\overrightarrow{u} - \overrightarrow{v} = \overrightarrow{u} + (-\overrightarrow{v}).$$

Definition: If we place the initial point of a vector $\overrightarrow{a}$ at the origin of a rectangular coordinate system, then the terminal point of $\overrightarrow{a}$ has coordinates of the form (a_1, a_2) or (a_1, a_2, a_3) depending on whether our coordinate system is two- or three- dimensional. These coordinates are called the **components** of $\overrightarrow{a}$ and we write

$$\overrightarrow{a} = < a_1,\ a_2 > \text{ or } \overrightarrow{a} = < a_1,\ a_2,\ a_3 >.$$

Definition: The particular representation $\overrightarrow{OP}$ from the origin to the point P is called the **position vector** of the point P.

Definition: Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\overrightarrow{v}$ with representation $\overrightarrow{AB}$ is

$$\overrightarrow{v} = < x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 >.$$

Definition: The **magnitude** or **length** of the vector $\overrightarrow{v}$ is the length of any of its representations.

Notation: The **length** is denoted by $|\overrightarrow{v}|$.

Formula: The **length** of the two-dimensional vector $\overrightarrow{a} = < a_1,\ a_2 >$ is

$$|\ \overrightarrow{a}\ | = \sqrt{a_1^2 + a_2^2}.$$

Formula: The **length** of the three-dimensional vector $\overrightarrow{a} = < a_1,\ a_2,\ a_3 >$ is

$$|\ \overrightarrow{a}\ | = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

Formula: If $\overrightarrow{a} = < a_1,\ a_2 >$ and $\overrightarrow{b} = < b_1,\ b_2 >$, then

$$\begin{aligned} \overrightarrow{a} + \overrightarrow{b} &= < a_1 + b_1,\ a_2 + b_2 > \\ \overrightarrow{a} - \overrightarrow{b} &= < a_1 - b_1,\ a_2 - b_2 > \\ c\overrightarrow{a} &= < ca_1,\ ca_2 > \end{aligned}$$

Similarly, for three-dimensional vectors,

$$< a_1, a_2, a_3 > + < b_1, b_2, b_3 > = < a_1 + b_1, a_2 + b_2, a_3 + b_3 >$$
$$< a_1, a_2, a_3 > - < b_1, b_2, b_3 > = < a_1 - b_1, a_2 - b_2, a_3 - b_3 >$$
$$c < a_1, a_2, a_3 > = < ca_1, ca_2, ca_3 >$$

Properties of Vectors: If $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ in V_n and c and d are scalars, then:
1. $\overrightarrow{a} + \overrightarrow{b} = \overrightarrow{b} + \overrightarrow{a}$
2. $\overrightarrow{a} + (\overrightarrow{b} + \overrightarrow{c}) = (\overrightarrow{a} + \overrightarrow{b}) + \overrightarrow{c}$
3. $\overrightarrow{a} + \overrightarrow{0} = \overrightarrow{a}$
4. $\overrightarrow{a} + (-\overrightarrow{a}) = \overrightarrow{0}$
5. $c(\overrightarrow{a} + \overrightarrow{b}) = c\overrightarrow{a} + c\overrightarrow{b}$
6. $(c+d)\overrightarrow{a} = c\overrightarrow{a} + d\overrightarrow{a}$
7. $(cd)\overrightarrow{a} = c(d\overrightarrow{a})$
8. $1\overrightarrow{a} = \overrightarrow{a}$

Definition: The **standard basis vectors** are $\overrightarrow{i}$, $\overrightarrow{j}$, and $\overrightarrow{k}$, where

$$\overrightarrow{i} = < 1, 0, 0 > \qquad \overrightarrow{j} = < 0, 1, 0 > \qquad \overrightarrow{k} = < 0, 0, 1 >$$

Exercise 1. Find $\overrightarrow{a} + \overrightarrow{b}$, $2\overrightarrow{a} + 3\overrightarrow{b}$, $|\overrightarrow{a}|$, $|\overrightarrow{a} - \overrightarrow{b}|$.
(a) $\overrightarrow{a} = \overrightarrow{i} + \overrightarrow{j}$, $\overrightarrow{b} = 2\overrightarrow{i} - 5\overrightarrow{j}$
(b) $\overrightarrow{a} = 2\overrightarrow{i} - 5\overrightarrow{j}$, $\overrightarrow{b} = 3\overrightarrow{i} - \overrightarrow{j}$

Class Exercise 1. Find $\overrightarrow{a} + \overrightarrow{b}$, $2\overrightarrow{a} + 3\overrightarrow{b}$, $|\overrightarrow{a}|$, $|\overrightarrow{a} - \overrightarrow{b}|$.
(a) $\overrightarrow{a} = 4\overrightarrow{i} + \overrightarrow{j}$, $\overrightarrow{b} = \overrightarrow{i} - 2\overrightarrow{j}$ (#20)
(b) $\overrightarrow{a} = 2\overrightarrow{i} - 4\overrightarrow{j} + 4\overrightarrow{k}$, $\overrightarrow{b} = 2\overrightarrow{j} - \overrightarrow{k}$ (#22)

Definition: A **unit vector** is a vector whose length is 1.

Exercise 2. Find a unit vector $\overrightarrow{u}$ in the direction of the vector from $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$. (Hass Sec 12.2 Example 4)

Exercise 3. Find the unit vector $\overrightarrow{u}$ that has the same direction as $3\overrightarrow{i} - 4\overrightarrow{j}$. (Swokowski Sec 14.1 Example 6)

Class Exercise 2. Find a unit vector that has the same direction as the given vector $< -4, 2, 4 >$. (#24)

Exercise 4. A 75-N weight is suspended by two wires, as shown in the figure in the video. Find the forces $\overrightarrow{F}_1$ and $\overrightarrow{F}_2$ acting on both wires. (Hass Sec 12.2 Example 9)

Class Exercise 3. Ropes 3 m and 5 m in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes, fastened at different heights, make angles of 52° and 40° with the horizontal. Find the tension in each and the magnitude of each tension. (#36)

Homework: 3-35 (every 4th)