Section 12.3

Definition: If $\vec{a} = < a_1, a_2, a_3 >$ and $\vec{b} = < b_1, b_2, b_3 >$, then the dot product of $\vec{a}$ and $\vec{b}$ is the number of $\vec{a} \cdot \vec{b}$ given by

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

Exercise 1. Find $\vec{a} \cdot \vec{b}$.
(a) $\vec{a} = < 2, 4, -3 >$, $\vec{b} = < -1, 5, 2 >$.
(b) $\vec{a} = 3\vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = 4\vec{i} + 5\vec{j} - 2\vec{k}$ (Swokowski Sec 14.3 Example 1)

Class Exercise 1. Find $\vec{a} \cdot \vec{b}$. (#2-8 even)
(a) $\vec{a} = < -2, 3 >$, $\vec{b} = < 0.7, 1.2 >$
(b) $\vec{a} = < 6, -2, 3 >$, $\vec{b} = < 2, 5, -1 >$
(c) $\vec{a} = < p, -p, 2p >$, $\vec{b} = < 2q, q, -q >$
(d) $\vec{a} = 3\ \vec{i} + 2\ \vec{j} - \vec{k}$, $\vec{b} = 4\ \vec{i} + 5\ \vec{k}$

Properties of the Dot Product: If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors in V_3 and c is a scalar, then
1. $\vec{a} \cdot \vec{a} = |\ \vec{a}\ |^2$
2. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
3. $\vec{a} \cdot (\ \vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
4. $(c\vec{a}) \cdot \vec{b} = c(\vec{a} \cdot \vec{b}) = \vec{a} \cdot (c\,\vec{b})$
5. $\vec{0} \cdot \vec{a} = 0$

Theorem: If θ is the angle between the vectors $\vec{a}$ and $\vec{b}$, then

$$\vec{a} \cdot \vec{b} = |\ \vec{a}\ |\ |\ \vec{b}\ | \cos\theta.$$

Corollary: If θ is the angle between the nonzero vectors $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}.$$

Exercise 2. Find the angle between the vectors $\vec{a} = < 4, -3, 1 >$ and $\vec{b} = < -1, -2, 2 >$. (Swok Sec 14.3 Example 2)

Exercise 3. Find the angle between $\vec{u} = \vec{i} - 2\vec{j} - 2\vec{k}$ and $\vec{v} = 6\vec{i} + 3\vec{j} + 2\vec{k}$ (Hass Sec 12.3 Ex 2).

Exercise 4. Find the angle θ in the triangle ABC determined by the vertices A = (0,0), B = (3,5), and C = (5,2). (Hass Sec 12.3 Ex 3)

Class Exercise 2. Find the angle between the vectors.
(a) $\vec{a} = < -2, 5 >$ and $\vec{b} = < 5, 12 >$
(b) $\vec{a} = < 4, 0, 2 >$ and $\vec{b} = < 2, -1, 0 >$
(c) $\vec{a} = \vec{i} + 2\vec{j} - 2\vec{k}$ and $\vec{b} = 4\vec{i} - 3\vec{k}$ (#16,18,20)

Class Exercise 3. Find, correct to the nearest degree, the three angles of the triangle with given vertices $A(1, 0, -1)$, $B(3, -2, 0)$, and $C(1, 3, 3)$. (#22)

Definition: Two nonzero vectors $\vec{a}$ and $\vec{b}$ are called **perpendicular** or **orthogonal** if the angle between them is 90°.

Theorem: Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if $\vec{a} \cdot \vec{b} = 0$.

Exercise 5. Prove that the vectors are orthogonal.
(a) $\vec{i}$, $\vec{j}$
(b) $3\vec{i} - 7\vec{j} + 2\vec{k}$, $10\vec{i} + 4\vec{j} - \vec{k}$ (Swok Sec 14.3 Exercise 3)

Class Exercise 4. Determine whether the given vectors are orthogonal, parallel, or neither.
(a) $\overrightarrow{u} = < -3, 9, 6 >$, $\overrightarrow{v} = < 4, -12, -8 >$
(b) $\overrightarrow{u} = \overrightarrow{i} - \overrightarrow{j} + 2\overrightarrow{k}$, $\overrightarrow{v} = 2\overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$
(c) $\overrightarrow{u} = < a, b, c >$, $\overrightarrow{v} = < -b, a, 0 >$ (#24)

Definition: The **direction angles** of a nonzero vector $\overrightarrow{a} = < a_1, a_2, a_3 >$ are the angles α, β, and γ that $\overrightarrow{a}$ makes with the positive $x-$, $y-$, and $z-$axes, respectively.

Definition: The **direction** cosines of these direction angles are $\cos\alpha$, $\cos\beta$, and $\cos\gamma$.

Here are some formulas:

$$\cos\alpha = \frac{a_1}{|\overrightarrow{a}|} \qquad \cos\beta = \frac{a_2}{|\overrightarrow{a}|} \qquad \cos\gamma = \frac{a_3}{|\overrightarrow{a}|}$$

Exercise 6. Find the direction cosines and direction angles of the vector: $< 1, 2, 3 >$.
(Stewart Ex 5)

Class Exercise 5. Find the direction cosines and direction angles of the vector. (Give the direction angles correct to the nearest degree.)
(a) $< 6, 3, -2 >$
(b) $\frac{1}{2}\overrightarrow{i} + \overrightarrow{j} + \overrightarrow{k}$ (#34, 36)

Definition: Suppose $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are representations of two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ with same initial point P. If S is the foot of the perpendicular from R to the line containing $\overrightarrow{PQ}$, then the vector with representation $\overrightarrow{PS}$ is called the **vector projection** of $\overrightarrow{b}$ onto $\overrightarrow{a}$.

Notation: The notation for the vector projection of $\overrightarrow{v}$ onto $\overrightarrow{v}$ is $\text{proj}_{\overrightarrow{a}}\overrightarrow{b}$.

Definition: The **scalar projection** of $\overrightarrow{b}$ onto $\overrightarrow{a}$ is defined to be the signed magnitude of the vector projection, which is the number $|\overrightarrow{b}|\cos\theta$, where θ is the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

Notation: The notation for the scalar projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$ is $\text{comp}_{\overrightarrow{a}}\overrightarrow{b}$.

Formula: The scalar projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$:

$$\text{comp}_{\overrightarrow{a}}\overrightarrow{b} = \frac{(\overrightarrow{a}\cdot\overrightarrow{b})}{|\overrightarrow{a}|}$$

Formula: The vector projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$:

$$\text{proj}_{\overrightarrow{a}}\overrightarrow{b} = \frac{\overrightarrow{a}\cdot\overrightarrow{b}}{|\overrightarrow{a}|^2}\overrightarrow{a}$$

Exercise 7. Find the vector projection of $\overrightarrow{u} = 6\overrightarrow{i} + 3\overrightarrow{j} + 2\overrightarrow{k}$ onto $\overrightarrow{v} = \overrightarrow{i} - 2\overrightarrow{j} - 2\overrightarrow{k}$ and the scalar projection of $\overrightarrow{u}$ in the direction of $\overrightarrow{v}$. (Hass Sec 12.3 Ex 5)

Exercise 8. Find the vector projection of a force $\overrightarrow{F} = 5\overrightarrow{i} + 2\overrightarrow{j}$ onto $\overrightarrow{v} = \overrightarrow{i} - 3\overrightarrow{j}$ and the scalar component of $\overrightarrow{F}$ in the direction of $\overrightarrow{v}$. (Hass Sec 12.3 Ex 6)

Class Exercise 6. Find the scalar and vector projections of $\overrightarrow{b}$ onto $\overrightarrow{a}$.
(a) $\overrightarrow{a} = < 1, 4 >$, $\overrightarrow{b} = < 2, 3 >$
(b) $\overrightarrow{a} = < -2, 3, -6 >$, $\overrightarrow{b} = < 5, -1, 4 >$
(c) $\overrightarrow{a} = \overrightarrow{i} + \overrightarrow{j} + \overrightarrow{k}$, $\overrightarrow{b} = \overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$ (#40,42,44)

Homework: 3-55 (every 4th)