Section 12.6

<u>Definition</u>: The curves of intersection of the surface with planes parallel to the coordinate planes are called **<u>traces</u>**.

Exercise 1. Describe and sketch the surface: $y = 4x^2 + 9z^2$. (Metzler)

<u>Definition</u>: A **<u>cylinder</u>** is a surface that consists of all lines that are parallel to a given line and pass through a given plane curve.

Exercise 2. Sketch the surface: $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Class Exercise 1. Describe and sketch the surface. (#4, 6, 8)
(a) $4x^2 + y^2 = 4$ (b) $y = z^2$ (c) $z = \sin y$.

<u>Definition</u>: A **<u>quadric surface</u>** is a graph of a second-degree equation in three variables x, y, and z. The most general such equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0,$$

where A, B, C,........, J are constants.

<u>Definition</u>: An **<u>ellipsoid</u>** is a surface where all the traces are ellipses. Here is the equation of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Exercise 3. Draw an ellipsoid. (Hutchings 1.4.3)

<u>Definition</u>: An **<u>elliptic paraboloid</u>** is a surface where all the horizontal traces are ellipses and all the vertical traces are parabolas. Here is the equation of an elliptic paraboloid:

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}.$$

Exercise 4. Draw an elliptic paraboloid. (Hutchings 1.4.4)

<u>Definition</u>: A **<u>hyperbolic paraboloid</u>** is a surface where all the horizontal traces are hyperbolas and all the vertical traces are parabolas. Here is the equation of a hyperbolic paraboloid:

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Exercise 5. Draw a hyperbolic paraboloid. (Hutchings 1.4.5)

<u>Definition</u>: A **<u>cone</u>** is a surface where all the horizontal traces are ellipses and vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$. Here is the equation of a cone:

$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Exercise 6. Draw a cone. (Hutchings 1.4.4)

<u>Definition</u>: A **<u>hyperboloid of one sheet</u>** is a surface where all the horizontal traces are ellipses and vertical traces are hyperbolas. Here is an equation of a hyperboloid of one sheet:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

<u>Definition</u>: A **<u>hyperboloid of two sheets</u>** is a surface where horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$ and vertical traces are hyperbolas. The equation of a hyperboloid of two sheets:

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Exercise 7. Draw a hyperboloid of one sheet and a hyperboloid of two sheets. (Hutchings 1.4.4)

Class Exercise 2. Use traces to sketch and identify the surface. (#12-20 even)
(a) $9x^2 - y^2 + z^2 = 0$ (b) $25x^2 + 4y^2 + z^2 = 100$ (c) $4x^2 + 9y^2 + z = 0$
(d) $4x^2 - 16y^2 + z^2 = 16$ (e) $x = y^2 - z^2$

Exercise 8. Identify and sketch the surface $4x^2 - y^2 + 2z^2 + 4 = 0$. (Stew Sec 12.6 Ex 7)

Exercise 9. Classify the quadratic surface $x^2 + 2z^2 - 6x - y + 10 = 0$. (Stew Sec 12.6 Ex 8)

Class Exercise 3. Reduce the equation to one of the standard forms, classify the surface, and sketch it. (#30-36 even).
(a) $4x^2 - y + 2z^2 = 0$ (b) $y^2 = x^2 + 4z^2 + 4$
(c) $4y^2 + z^2 - x - 16y - 4z + 20 = 0$
(d) $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$

Homework: 3, 7, 13-37 (every 4th), 45, 47