Section 13.1

Definition: A **vector-valued function** is simply a function whose domain is a set of real numbers and whose range is a set of vectors.

Definition: If $f(t)$, $g(t)$, and $h(t)$ are the components of the vector $\vec{r}(t)$, then f, g, and h are real-valued functions called the component functions of $\vec{r}$ and we can write

$$\vec{r}(t) = < f(t), g(t), h(t) >.$$

Definition: The limit of a vector function $\vec{r}(t)$ is defined by taking the limits of its component functions as follows. If $\vec{r} = < f(t), g(t), h(t) >$, then

$$\lim_{t\to a} \vec{r}(t) = < \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) >$$

provided the limits of the component functions exist.

Exercise 1. Find $\lim_{t\to 2} (t^2 \vec{i} + 3t\vec{j} + 5\vec{k})$. (Swok 15.2 Illustration)

Class Exercise 1. Find the limit. (#4,6)
(a) $\lim_{t\to 1} ((\frac{t^2-t}{t-1})\vec{i} + \sqrt{t+8}\vec{j} + \frac{\sin(\pi t)}{\ln t}\vec{k})$
(b) $\lim_{t\to\infty} < te^{-t}, \frac{t^3+t}{2t^3-1}, t \sin \frac{1}{t} >$

Definition: A vector function $\vec{r}$ is **continuous at** a if

$$\lim_{t\to a} \vec{r}(t) = \vec{r}(a).$$

Definition: Suppose that f, g, and h are continuous real-valued functions on an interval I. Then the set C of all points (x, y, z) in space, where

$$x = f(t) \qquad y = g(t) \qquad z = h(t)$$

and t varies throughout the interval I, is called the **space curve**. The equations are called **parametric equations of C** and t is called a parameter.

Exercise 2. Let $\vec{r}$(t) = a cos $t\,\vec{i}$ + a sin $t\,\vec{j}$ + $bt\,\vec{k}$ for t ≥ 0, and positive constants a and b. Sketch the curve C determined by $\vec{r}(t)$, and indicate the orientation. (Swok Sec 15.1 Ex 4)

Exercise 3. Let $\vec{r}(t) = 2\text{t}\,\vec{i} + (8 - 2t^2)\vec{j}$ for $-2 \leq t \leq 2$. Sketch the curve C determined by $\vec{r}(t)$, and indicate the orientation. (Swok Sec 15.1 Ex 5)

Exercise 4. Let $\vec{r}(t) = (\frac{1}{2}t\cos t)\vec{i} + (\frac{1}{2}t\sin t)\vec{j}$ for $0 \leq t \leq 3\pi$. (Swok Sec 15.1 Ex 6)
(a) Sketch the curve determined by $\vec{r}(t)$, and indicate the orientation.
(b) Sketch $\vec{r}(6)$.

Class Exercise 2. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases. (#8-14 even)
(a) $\vec{r}(t) = < t^3, t^2 >$ (b) $\vec{r}(t) = < \sin \pi t, t, \cos \pi t >$
(c) $\vec{r}(t) = t^2\vec{i} + t\vec{j} + 2\vec{k}$ (d) $\vec{r}(t) = \cos t\,\vec{i} - \cos t\,\vec{j} + \sin t\,\vec{k}$

Exercise 5. Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$. (Stew 13.1 Ex 6)

Class Exercise 3. Find a vector function that represents the curve of intersection of the two surfaces. (#40, 42, 44).
(a) The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$
(b) The paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$
(c) The semiellipsoid $x^2 + y^2 + 4z^2 = 4$, $y \geq 0$, and the cylinder $x^2 + z^2 = 1$

Homework: 3, 7, 13, 19, 21, 27, 35, 39, 49-53 ODD