Section 13.2

Definition: The derivative $\vec{r}\,'$ of a vector function $\vec{r}$ is defined in much the same way as for real-valued functions:

$$\frac{d\vec{r}}{dt} = \vec{r}\,'(t) = \lim_{h\to 0} \frac{\vec{r}(t+h)-\vec{r}(t)}{h}.$$

Theorem: If $\vec{r}(t) = <f(t),\ g(t),\ h(t)> = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$, where f, g, and h are differentiable functions, then

$$\vec{r}\,'(t) = < f'(t), g'(t), h'(t) > = f'(t)\vec{i} + g'(t)\vec{j} + h'(t)\vec{k}.$$

Exercise 1. Compute the derivatives of the following functions. (Briggs Sec 11.6 Ex 1)
(a) $\vec{r}(t) = < t^3, 3t^2, t^3/6 >$ (b) $\vec{r}(t) = e^{-t}\vec{i} + 10\sqrt{t}\,\vec{j} + 2\cos 3t\,\vec{k}$

Class Exercise 1. Find the derivative of the vector function. (#10, 12, 14)
(a) $\vec{r}(t) = < \tan t, \sec t, 1/t^2 >$ (b) $\vec{r}(t) = \frac{1}{1+t}\vec{i} + \frac{t}{1+t}\vec{j} + \frac{t^2}{1+t}\vec{k}$
(c) $\vec{r}(t) = at\cos 3t\ \vec{i} + b\sin^3 t\ \vec{j} + c\cos^3 t\ \vec{k}$

Definition: The vector $\vec{r}\,'(t)$ is called the **tangent vector** to the curve defined by $\vec{r}$ at the point P, provided that $\vec{r}\,'(t) \neq 0$.

Exercise 2. Let $\vec{r}(t) = 2t\ \vec{i} + (t^2-4)\vec{j}$ for $-2 \le t \le 3$. Find $\vec{r}\,'(t)$ and sketch $\vec{r}(1)$ and $\vec{r}\,'(1)$. (Swok Sec 15.2 Ex 2)

Class Exercise 2. (i) Sketch the plane curve with the given vector equation. (ii) Find $\vec{r}\,'(t)$. (iii) Sketch the position vector $\vec{r}(t)$ and the tangent vector $\vec{r}\,'(t)$ for the given value of t.
(a) $\vec{r}(t) = < t^2, t^3 >, t = 1$ (b) $\vec{r}(t) = e^t\vec{i} + e^{-t}\vec{j}, t = 0$
(c) $\vec{r}(t) = (1+\cos t)\vec{i} + (2+\sin t)\vec{j}, t = \pi/6$ (#4,6,8)

Definition: The **unit tangent vector** is

$$T(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}.$$

Exercise 3. Find the unit tangent vectors for the following curves. (Briggs Sec 11.6 Ex 2)
(a) $\vec{r}(t) = < t^2, 4t, 4\ln t >$, for $t > 0$ (b) $\vec{r}(t) = < 10, 3\cos t, 3\sin t >$, for $0 \le t \le 2\pi$

Class Exercise 3. Find the unit tangent vector $\vec{T}(t)$ at the point with the given value of the parameter t.(#18, 20)
(a) $\vec{r}(t) = <t^3+3t, t^2+1, 3t+4>, t = 1$ (b) $\vec{r}(t) = \sin^2 t\,\vec{i} + \cos^2 t\,\vec{j} + \tan^2 t\,\vec{k}, t = \pi/4$.

Definition: The **tangent line** to C at P is defined to be the line through P parallel to the tangent vector $\vec{r}\,'(t)$.

Exercise 4. Let C be the curve with parametric equations $x = t$, $y = t^2$, $z = t^3$, $t \ge 0$. Find parametric equations for the tangent line to C at the point corresponding to $t = 2$. (Swok Sec 15.2 Ex 3)

Class Exercise 4. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point. (#24,26)
(a) $x = e^t, y = te^t, z = te^{t^2}$ (1,0,0) (b) $x = \sqrt{t^2+3}, y = \ln(t^2+3), z = t$ (2, ln 4, 1)

Here is a formula for integrating a vector function

$$\int_a^b \vec{r}(t)\,dt = \left(\int_a^b f(t)\,dt\right)\vec{i} + \left(\int_a^b g(t)\,dt\right)\vec{j} + \left(\int_a^b h(t)\,dt\right)\vec{k}$$

Exercise 5. Find $\int_0^2 \vec{r}(t)\,dt$ if $\vec{r}(t) = 12t^3\vec{i} + 4e^{2t}\vec{j} + (t+1)^{-1}\vec{k}$. (Swok Sec 15.2 Ex 4)

Exercise 6. Evaluate the integral: $\int_0^\pi (\vec{i} + 3\cos\frac{t}{2}\vec{j} - 4t\vec{k})\,dt$. (Briggs Sec 11.6 Ex 7)

Class Exercise 5. Evaluate the integral. (#36, 38, 40)
(a) $\int_0^1 (\frac{4}{1+t^2}\vec{j} + \frac{2t}{1+t^2}\vec{k})\,dt$ (b) $\int_1^2 (t^2\vec{i} + t\sqrt{t-1}\,\vec{j} + t\sin(\pi t)\vec{k})\,dt$
(c) $\int (te^{2t}\vec{i} + \frac{t}{1-t}\vec{j} + \frac{1}{\sqrt{1-t^2}}\vec{k})\,dt$.

Homework: 3-19 (every 4th), 25, 29, 37, 41