Section 13.4

Definition: Suppose that the position vector of a particle is $\vec{r}(t)$. The **velocity vector $\vec{v}(t)$** at time t is

$$\vec{v}(t) = \lim_{h\to 0} \frac{\vec{r}(t+h)-\vec{r}(t)}{h} = \vec{r}\,'(t).$$

Definition: The **speed** of the particle at time t is the magnitude of the velocity vector, that is,

$$|\ \vec{v}(t)\ |.$$

Definition: The **acceleration of a particle** is defined as the derivative of the velocity:

$$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t).$$

Exercise 1. The position vector of a point P moving in an xy-plane is

$$\vec{r}(t) = (t^2+t)\vec{i} + t^3\vec{j} \text{ for } 0 \le t \le 2.$$

(a) Find the velocity and acceleration of P at time t.
(b) Sketch the path C of the point, together with $\vec{v}(1)$ and $\vec{a}(1)$. (Swok Sec 15.3 Ex 1)

Class Exercise 1. Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t. (#4,6,8)
(a) $\vec{r}(t) = <2-t,\, 4\sqrt{t}>$, $t=1$ (b) $\vec{r}(t) = e^t\vec{i} + e^{2t}\vec{j}$, $t=0$
(c) $\vec{r}(t) = t\,\vec{i} + 2\cos t\ \vec{j} + \sin t\ \vec{k}$, $t=0$.

Ideal Projectile Motion Equation:

The parametric equations of the trajectory of a projectile are:

$$\vec{r}(t) = (v_0 \cos\alpha)t\,\vec{i} + ((v_0 \sin\alpha)t - \tfrac{1}{2}gt^2)\vec{j},$$

where v_0 is the initial velocity, α is the angle at which the projectile is launched, and g is the gravitational constant.

The horizontal distance d traveled by the projectile is

$$d = \frac{v_0^2 \sin 2\alpha}{g}.$$

Exercise 2. A projectile is fired from the origin over horizontal ground at an initial speed of 500 m/sec and a launch angle of 60°. Where will the projectile be 10 sec later?

Class Exercise 2. A gun is fired with angle of elevation 30°. What is the muzzle speed if the maximum height of the shell is 500 m? (#26)

Class Exercise 3. A batter hits a baseball 3 ft above the ground toward the center field fence, which is 10 ft high and 400 ft from home plate. The ball leaves the bat with speed 115 ft/s at an angle 50° above the horizontal. Is it a home run? (In other words, does the ball clear the fence?) (#28)

Formula: The tangential component of acceleration is: $a_T = \frac{\vec{r}\,'(t)\cdot\vec{r}\,''(t)}{|\vec{r}\,'(t)|}$

Formula: The normal component of acceleration is: $a_N = |\frac{\vec{r}\,'(t) X \vec{r}\,''(t)|}{|\vec{r}\,'(t)|}$.

Exercise 3. The position vector of a moving point at time t is $\vec{r}(t) = t^2\vec{i} + t^2\vec{j} + t^3\vec{k}$ for $1 \le t \le 4$. Find the tangential and normal components of acceleration at time t. (Stew Ex 7)

Class Exercise 4. Find the tangential and normal components of the acceleration vector. (#38, 40, 42)
(a) $\vec{r}(t) = (1+t)\vec{i} + (t^2-2t)\vec{j}$ (b) $\vec{r}(t) = t\,\vec{i} + t^2\vec{j} + 3t\,\vec{k}$
(c) $\vec{r}(t) = t\,\vec{i} + \cos^2 t\,\vec{j} + \sin^2 t\,\vec{k}$

Homework: 3-27 (every 4th), 37-41 ODD