Section 14.1

Definition: A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{\ f(x, y) \mid (x, y) \in D\ \}$.

Exercise 1. Let $f(x, y) = \frac{xy-5}{2\sqrt{y-x^2}}$. Sketch the domain D of f. (Swok Sec 16.1 Ex 1)

Exercise 2. Find the domain of the function $g(x, y) = \sqrt{4 - x^2 - y^2}$. (Briggs Sec 12.2 Ex 1)

Class Exercise 1. Find and sketch the domain of the function. (#14-20 even)
(a) $f(x, y) = \sqrt{xy}$ (b) $f(x, y) = \sqrt{x^2 - y^2}$ (c) $f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$
(d) $f(x, y) = \arcsin(x^2 + y^2 - 2)$

Definition: If f is a function of two variables with domain D, then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^2$ such that $z = f(x, y)$ and (x, y) is in D.

Exercise 3. Find the domain and range of the following functions. Then sketch a graph.
(Briggs Sec 12.2 Ex 2)
(a) $f(x, y) = 2x + 3y - 12$ (b) $g(x, y) = x^2 + y^2$ (c) $h(x, y) = \sqrt{1 + x^2 + y^2}$

Class Exercise 2. Sketch the graph of the function. (#24-30 even)
(a) $f(x, y) = 2 - x$ (b) $f(x, y) = e^{-y}$
(c) $f(x, y) = 1 + 2x^2 + 2y^2$ (d) $f(x, y) = \sqrt{4x^2 + y^2}$

Definition: The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f).

Exercise 4. Graph $f(x, y) = 100 - x^2 - y^2$ and plot the level curves $f(x, y) = 0$, $f(x, y) = 51$, $f(x, y) = 75$ in the domain of f in the plane. (Hass Sec 14.1 Ex 5)

Exercise 5. If $f(x, y) = y^2 - x^2$, sketch some level curves of f. (Swok Sec 16.1 Ex 4)

Exercise 6. Find and sketch the level curves of the following surfaces: (a) $f(x, y) = y - x^2 - 1$ and (b) $f(x, y) = e^{-x^2-y^2}$. (Briggs Sec 12.2 Ex 3)

Class Exercise 3. Draw a contour map of the function showing several level curves.
(#44-50 even) (a) $f(x, y) = x^3 - y$ (b) $f(x, y) = \ln(x^2 + 4y^2)$
(c) $f(x, y) = y \sec x$ (d) $f(x, y) = \frac{y}{x^2+y^2}$

Definition: A **function of three variables**, f, is a rule that assigns to each ordered triple (x, y, z) in a domain $D \subset \mathbb{R}^3$ a unique real number denoted by $f(x, y, z)$.

Class Exercise 4. Find and sketch the domain of the function: $f(x, y, z) = \ln(16 - 4x^2 - 4y^2 - z^2)$ (#22)

Homework: 3-19 (every 4th), 25-45 (every 4th)