Section 14.2

Let's review limits from single variable calculus.

Definition: Suppose $f(x)$ is defined on an open interval about c, *except possibly at c itself.* If $f(x)$ is arbitrarily close to L as we like for all x sufficiently close to c, we say that f approaches the **limit** L as x approaches c, and write

$$\lim_{x\to c} f(x) = L,$$

which is read "the limit of $f(x)$ as x approaches c is L."

Definition: We use the notation

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

to indicate that the values of $f(x,y)$ approach the number L as the point (x,y) approaches the point (a,b) along any path that stays within the domain of f.

Exercise 1. Find (a) $\lim_{(x,y)\to(2,-3)} (x^3 - 4xy^2 + 5y - 7)$
(b) $\lim_{(x,y)\to(3,4)} \frac{x^2-y^2}{\sqrt{x^2+y^2}}$ (Swok Sec 16.2 Ex 1)

Two-Path Rule: If two different paths to a point $P(a,b)$ produce two different limiting values for f, then $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist.

Exercise 2. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist. (Swok Sec 16.2 Ex 2)

Exercise 3. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist. (Swok Sec 16.2 Ex 3)

Exercise 4. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ does not exist. (Swok Sec 16.2 Ex 4)

Class Exercise 1. Find the limit, if it exists, or show that the limit does not exist. (#6-18 even)
(a) $\lim_{(x,y)\to(1,-1)} e^{-xy}\cos(x+y)$ (b) $\lim_{(x,y)\to(1,0)} \ln(\frac{1+y^2}{x^2+xy})$
(c) $\lim_{(x,y)\to(0,0)} \frac{5y^4\cos^2 x}{x^4+y^4}$ (d) $\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2}$
(e) $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$ (f) $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2}$ (g) $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$

Definition: The notation

$$\lim_{(x,y,z)\to(a,b,c)} f(x,y,z) = L$$

means that the values of $f(x,y,z)$ approach the number L as the point (x,y,z) approaches the point (a,b,c) along any path in the domain of f.

Class Exercise 2. Find the limit, if it exists, or show that the limit does not exist (#20,22)
(a) $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz}{x^2+y^2+z^2}$ (b) $\lim_{(x,y,z)\to(0,0,0)} \frac{yz}{x^2+4y^2+9z^2}$

Definition: A function f of two variables is called **continuous at** (a,b) if

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b).$$

We say that f is **continuous on** D if f is continuous at every point (a,b) in D.

Exercise 5. If $h(x,y) = e^{x^2+5xy+y^2}$, show that h is continuous at every pair (a,b).
(Swok Sec 16.2 Ex 7)

Exercise 6. Show that $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & \text{if}(x,y) \neq (0,0) \\ 0, & \text{if}(x,y) = (0,0) \end{cases}$ is continuous at every point except at the origin. (Hass Sec 14.2 Ex 5)

Exercise 7. Determine the points at which the following functions are continuous.
(a) $h(x,y) = \ln(x^2+y^2+4)$ (b) $h(x,y) = e^{x/y}$ (Briggs Sec 12.3 Ex 5)

Class Exercise 3. Determine the set of points at which the function is continuous. (#30-36 even)
(a) $F(x,y) = \cos\sqrt{1+x-y}$ (b) $H(x,y) = \frac{e^x+e^y}{e^{xy}-1}$
(c) $G(x,y) = \tan^{-1}((x+y)^{-2})$ (d) $f(x,y,z) = \sqrt{y-x^2}\,\ln z$

Homework: 5-29 (every 4th), 37-49 (every 4th)