Section 14.8

Exercise 1. Find the point $p(x, y, z)$ on the plane $2x + y - z - 5 = 0$ that is closest to the origin. (Hass Sec 14.8 Ex 1)

Exercise 2. Find the points on the hyperbolic cylinder $x^2 - z^2 - 1 = 0$ that are closest to the origin. (Hass Sec 14.8 Ex 2)

Theorem: Suppose f and g are functions of two variables that have continuous first partial derivatives, and that $\nabla\, g \neq 0$ throughout a region of the xy-plane. If f has an extremum $f(x_0, y_0)$ subject to the constraint $g(x, y) = 0$, then there is a real number λ such that

$$\nabla\, f(x_0, y_0) = \lambda\, \nabla\, g(x_0, y_0).$$

Definition: The number λ is called a **Lagrange multiplier**.

Method of Lagrange Multipliers: To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$:

(a) Find all values of x, y, z, and λ such that

$$\nabla\, f(x, y, z) = \lambda\, \nabla\, g(x, y, z) \text{ and } g(x, y, z) = k.$$

(b) Evaluate f at all the points (x, y, z) that result from step (a). The largest of these values is the maximum value of f; the smallest is the minimum value of f.

Exercise 3. Find the extrema of $f(x, y) = xy$ if (x, y) is restricted to the ellipse $4x^2 + y^2 = 4$. (Swok Sec 16.9 Ex 1)

Exercise 4. Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$. (Hass Sec 14.8 Ex 4)

Class Exercise 1. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint. (#4,6)
(a) $f(x, y) = 3x + y$; $x^2 + y^2 = 10$ (b) $f(x, y) = e^{xy}$; $x^3 + y^3 = 16$

Exercise 5. Find the volume of the largest rectangular box with faces parallel to the coordinate planes that can be inscribed in the ellipsoid $16x^2 + 4y^2 + 9z^2 = 144$. (Swok Sec 16.9 Ex 2)

Exercise 6. If $f(x, y, z) = 4x^2 + y^2 + 5z^2$, find the point on the plane $2x + 3y + 4z = 12$ at which $f(x, y, z)$ has its least value. (Swok Sec 16.9 Ex 3)

Class Exercise 2. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint. $(\#8, 10, 12)$
(a) $f(x, y, z) = x^2 + y^2 + z^2$; $x + y + z = 12$ (b) $f(x, y, z) = x^2y^2z^2$; $x^2 + y^2 + z^2 = 1$
(c) $f(x, y, z) = x^4 + y^4 + z^4$; $x^2 + y^2 + z^2 = 1$

Homework: 1-15 ODD, 27-35 ODD