Section 15.2

Exercise 1. Evaluate: $\int_0^2 \int_{x^2}^{2x} (x^3 + 4y)\ dy\ dx$. (Swok Sec 17.1 Ex 3)

Exercise 2. Evaluate: $\int_1^3 \int_{\pi/6}^{y^2} 2y \cos x\ dx\ dy$. (Swok Sec 17.1 Ex 4)

Class Exercise 1. Evaluate the iterated integral. (#2,4,6)
(a) $\int_0^1 \int_{2x}^2 (x - y)\ dy\ dx$ (b) $\int_0^2 \int_y^{2y} xy\ dx\ dy$ (c) $\int_0^1 \int_0^{e^v} \sqrt{1 + e^v}\ dw\ dv$

Definition: A plane region D is said to be of **type I** if it lies between the graphs of two consecutive functions of x, that is,

$$D = \{\ (x, y) \mid a \leq x \leq b,\ g_1(x) \leq y \leq g_2(x)\ \}$$

Theorem: If f is continuous on a type I region D such that

$$D = \{\ (x, y) \mid a \leq x \leq b,\ g_1(x) \leq y \leq g_2(x)\ \},$$

then $\iint\limits_D f(x, y)\ dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\ dy\ dx$.

Definition: A plane region D is said to be of **type II** if it can be expressed as

$$D = \{\ (x, y) \mid c \leq y \leq d,\ h_1(y) \leq x \leq h_2(y)\ \},$$

where h_1 and h_2 are continuous.

Formula:

$$\iint\limits_D f(x, y)\ dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\ dx\ dy,$$

where D is a type II region.

Exercise 3. Given $\int_0^4 \int_{\sqrt{y}}^2 y \cos x^5\ dx\ dy$, reverse the order of integration and evaluate the resulting integral. (Swok Sec 17.1 Ex 7)

Exercise 4. Calculate

$$\iint_R (\sin x)/x\ dA,$$

where R is the triangle in the xy-plane bounded by the x-axis, the line $y = x$, and the line $x = 1$. (Hass Sec 15.2 Ex 2)

Class Exercise 2. Evaluate the double integral. (#8,10)
(a) $\iint\limits_D \frac{y}{x^5+1}\ dA$, $D = \{\ (x, y) \mid 0 \leq x \leq 1,\ 0 \leq y \leq x^2\ \}$
(b) $\iint\limits_D x^3\ dA$, $D = \{\ (x, y) \mid 1 \leq x \leq e,\ 0 \leq y \leq \ln x\ \}$

Exercise 5. Let R be the region in the xy-plane bounded by the graphs of $y = x^2$ and $y = 2x$. Evaluate $\iint_R (x^3 + 4y)\ dA$. (Swok Sec 17.1 Ex 5)

Exercise 6. Let R be the region bounded by the graphs of the equations $y = \sqrt{x}$, $y = \sqrt{3x - 18}$, and $y = 0$. If f is an arbitrary continuous function on R, express the double integral $\iint_R f(x, y)\ dA$ in terms of iterated integrals. (Swok Sec 17.1 Ex 6)

Class Exercise 3. Evaluate the double integral. (#18,20,22)
(a) $\iint\limits_D (x^2 + 2y)\ dA$, D is bounded by $y = x$, $y = x^3$, $x \geq 0$
(b) $\iint\limits_D xy^2\ dA$, D is enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$
(c) $\iint\limits_D 2xy\ dA$, D is the triangular region with vertices (0,0), (1,2), and (0,3).

Exercise 7. Find the volume V of the solid in the first octant bounded by the coordinate planes, the paraboloid $z = x^2 + y^2 + 1$, and the plane $2x + y = 2$. (Swok Sec 17.2 Ex 3)

Exercise 8. Find the volume V of the solid that lies in the first octant and is bounded by the three coordinate planes and the cylinders $x^2 + y^2 = 9$ and $y^2 + z^2 = 9$. (Swok Sec 17.2 Ex 4)

Class Exercise 4. Find the volume of the given solid. (#24, 26, 28, 30, 32)
(a) Under the surface $z = 1 + x^2y^2$ and above the region $x = y^2$ and $x = 4$
(b) Enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0$, $y = 1$, $y = x$, $z = 0$
(c) Bounded by the planes $z = x$, $y = x$, $x + y = 2$, and $z = 0$
(d) Bounded by the cylinders $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant
(e) Bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$

Exercise 9. Sketch the region of integration for the integral

$$\int_0^2 \int_{x^2}^{2x} (4x + 2)\ dy\ dx$$

and write an equivalent integral with the order of integration reversed. (Hass Sec 15.2 Ex 3)

Class Exercise 5. Sketch the region of integration and change the order of integration. (#44, 46, 48)
(a) $\int_0^2 \int_{x^2}^4 f(x,y)\ dy\ dx$ (b) $\int_{-2}^2 \int_0^{\sqrt{4-y^2}} f(x,y)\ dx\ dy$ (c) $\int_0^1 \int_{\arctan x}^{\pi/4} f(x,y)\ dy\ dx$

Class Exercise 6. Evaluate the integral by reversing the order of integration. (#50, 52, 54)
(a) $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\ dx\ dy$ (b) $\int_0^1 \int_x^1 e^{x/y}\ dy\ dx$ (c) $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\ dx\ dy$

Homework: 1, 5, 13, 17, 19, 25, 31-47 (every 4th), 55, 59, 61, 65