Section 15.5

Definition: Suppose S is a surface with equation $z = f(x, y)$, where f has continuous partial derivatives. The **surface area of S** is

$$A(S) = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \triangle T_{ij}.$$

Formula: The area of the surface with equation $z = f(x, y)$, $(x, y) \in D$, where f_x and f_y are continuous, is

$$A(S) = \iint_D \sqrt{[f_x(x, y)]^2 + [f_y(x, y)]^2 + 1}\, dA.$$

Formula with Alternative Notation

$$A(S) = \iint_D \sqrt{1 + (\tfrac{\partial z}{\partial x})^2 + (\tfrac{\partial z}{\partial y})^2}\, dA.$$

Exercise 1. Let R be the triangular region in the xy-plane with vertices (0,0,0), (0,1,0), and (1,1,0). Find the surface area of that portion of the graph $z = 3x + y^2$ that lies over R.
(Swok Sec 17.4 Ex 1)

Exercise 2. Find the surface area of the paraboloid $z = 4 - x^2 - y^2$ for $z \geq 0$.
(Swok Sec 17.4 Ex 2)

Class Exercise 1. Find the area of the surface. (#2,4,8,10,12)
(a) The part of the plane $2x + 5y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 9$
(b) The part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices (0,0), (0,1), and (2,1)
(c) The surface $z = \frac{2}{3}(x^{3/2} + y^{3/2})$, $0 \leq x \leq 1$, $0 \leq y \leq 1$
(d) The part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$
(e) The part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$

Homework: 3-13 ODD, 17(a)