Section 15.7

Definition: In the **cylindrical coordinate system**, a point P in three-dimensional space is represented by the ordered triple (r, θ, z), where r and θ are polar coordinates of the projection of P onto the xy-plane and z is the directed distance from the xy-plane to P.

Conversion From Cylindrical Coordinates to Rectangular Coordinates

$$x = r\cos\theta \quad y = r\sin\theta \quad z = z$$

Conversion From Rectangular Coordinates to Cylindrical Coordinates

$$r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x} \quad z = z$$

Exercise 1. (a) Plot the point $(2, 2\pi/3, 1)$ and find its rectangular coordinates.
(b) Find cylindrical coordinates of the point $(3, -3, -7)$. (Stew Sec 15.7 Ex 1)

Class Exercise 1. Plot the point whose cylindrical coordinates are given. Then find the rectangular coordinates of the point. (a) $(\sqrt{2},\ 3\pi/4,\ 2)$ (b) (1,1,1) (#2)

Class Exercise 2. Change from rectangular coordinates to cylindrical coordinates.
(a) $(2\sqrt{3}, 2, -1)$ (b) $(4, -3, 2)$ (#4)

Exercise 2. Change the equation $z^2 = x^2 + y^2$ to cylindrical coordinates, and sketch its graph. (Swok Sec 1.7 Ex 1)

Exercise 3. Identify and sketch the following sets in cylindrical coordinates.
(a) $Q = \{\ (r, \theta, z)\colon\ 1 \le r \le 3,\ z \ge 0\ \}$
(b) $S = \{\ (r, \theta, z)\colon\ z = 1 - r,\ 0 \le r \le 1\ \}$ (Briggs Sec 13.5 Ex 1)

Class Exercise 3. Describe in words the surface whose equation is given: $r = 5$. (#6)

Class Exercise 4. Identify the surface whose equation is given: $2r^2 + z^2 = 1$. (#8)

Class Exercise 5. Write the equations in cylindrical coordinates. (#10)
(a) $3x + 2y + z = 6$ (b) $-x^2 - y^2 + z^2 = 1$

Class Exercise 6. Sketch the solid described by the given inequalities:
$0 \le \theta \le \pi/2,\ r \le z \le 2$. (#12)

Formula for Triple Integration in Cylindrical Coordinates: Suppose that E is a type I region whose projection D onto the xy-plane is conveniently described in polar coordinates:

$$\iiint_E f(x, y, z)\, dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta,\ r\sin\theta,\ z)\ r\ dz\ dr\ d\theta$$

Exercise 4. Find the volume of the solid D between the cone $z = \sqrt{x^2 + y^2}$ and the inverted paraboloid $z = 12 - x^2 - y^2$. (Briggs Sec 13.5 Ex 4)

Exercise 5. A solid Q is bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$. The density at $P(x, y, z)$ is directly proportional to the square of the distance from the origin to P. Find its mass. (Swok Sec 17.7 Ex 4)

Class Exercise 7. Use cylindrical coordinates. (#18-24 even, 28)
(a) Evaluate $\iiint_E z\ dV$, where E is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.
(b) Evaluate $\iiint_E x\ dV$, where E is enclosed by the planes $z = 0$ and $z = x + y + 5$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.
(c) Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$
(d) Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$
(e) Find the mass of a ball B given by $x^2 + y^2 + z^2 \le a^2$ if the density at any point is proportional to its distance from the z-axis.

Homework: 1-13 ODD, 17-25 ODD, 27(a)