Section 15.8

Spherical Coordinates: The spherical coordinates of a point P are (ρ, θ, ϕ), where $\rho = |\ OP\ |$ is the distance from the origin to P, θ is the same angle as in cylindrical coordinates, and ϕ is the angle between the positive z-axis and the line segment OP.

Exercise 1. Plot the point $(4, \pi/3, \pi/6)$.

Class Exercise 1. Plot the points whose spherical coordinates are given. (#2)
(a) $(2, \pi/2, \pi/2)$ (b) $(4, -\pi/4, \pi/3)$

Conversion From Spherical to Rectangular Coordinates:

$$x = \rho \sin \phi \cos \theta \qquad y = \rho \sin \phi \sin \theta \qquad z = \rho \cos \phi$$

Conversion From Rectangular to Spherical Coordinates:

$$\rho^2 = x^2 + y^2 + z^2$$

Exercise 2. If a point P has spherical coordinates $(4, \pi/3, \pi/6)$, find rectangular points for P. (Swok Sec 17.8 Ex 1)

Class Exercise 2. Find the rectangular coordinates of the point whose spherical coordinates are given. (a) $(2, \pi/2, \pi/2)$ (b) $(4, -\pi/4, \pi/3)$ (#2)

Class Exercise 3. Change from rectangular to spherical coordinates. (#4)
(a) $(1, 0, \sqrt{3})$ (b) $(\sqrt{3}, -1, 2\sqrt{3})$

Class Exercise 4. Describe in words the surface whose equation is given: $\rho = 3$. (#6)

Exercise 3. Change the equation $\rho = 2 \sin \phi \cos \theta$ to rectangular coordinates, and describe its graph. (Swok Sec 17.8 Ex 3)

Class Exercise 5. Identify the surface whose equation is given:
$\rho^2(\sin^2\phi \sin^2\theta + \cos^2\phi) = 9$ (#8)

Exercise 4. Find an equation in spherical coordinates whose graph is the paraboloid:
$z = x^2 + y^2$. (Swok Sec 17.8 Ex 2)

Exercise 5. Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$
(Hass Sec 15.7 Ex 3)

Class Exercise 6. Write the equation in spherical coordinates:
(a) $x^2 - 2x + y^2 + z^2 = 0$ (b) $x + 2y + 3z = 1$. (#10)

Class Exercise 7. Sketch the solid described by the given inequalities. (#12,14)
(a) $1 \le \rho \le 2$, $0 \le \phi \le \pi/2$, $\pi/2 \le \theta \le 3\pi/2$ (b) $\rho \le 2$, $\rho \le \csc \phi$

Definition: In the spherical coordinate system the counterpart of a rectangular box is a **spherical wedge**

$$E = \{\ (\rho, \theta, \phi) \mid a \le \rho \le b,\ \alpha \le \theta \le \beta,\ c \le \phi \le d\ \},$$

where $\alpha \ge 0$ and $\beta - \alpha \le 2\pi$, and $d - c \le \pi$.

Formula for Triple Integration in Spherical Coordinates:

$$\iiint_E f(x, y, z)\, dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho \sin \phi \cos \theta,\ \rho \sin \phi \sin \theta,\ \rho \cos \phi)\ \rho^2 \sin \phi\ d\rho\ d\theta\ d\phi,$$

where E is a spherical wedge given by

$$E = \{\ (\rho, \theta, \phi) \mid a \le \rho \le b,\ \alpha \le \theta \le \beta,\ c \le \phi \le d\ \}.$$

Exercise 6. Evaluate $\iiint\limits_D (x^2+y^2+z^2)^{-3/2}\,dV$, where D is the region in the first octant between two spheres of radius 1 and 2 centered at the origin. (Briggs Sec 13.5 Ex 6)

Class Exercise 8. Use spherical coordinates. (#22-26 even)
(a) $\iiint_H (9 - x^2 - y^2)\,dV$, where H is the solid hemisphere $x^2 + y^2 + z^2 \le 9$, $z \ge 0$.
(b) $\iiint\limits_E y^2\,dV$, where E is the solid hemisphere $x^2 + y^2 + z^2 \le 9$, $y \ge 0$
(c) $\iiint\limits_E xyz\,dV$, where E lies between the spheres $\rho = 2$ and $\rho = 4$ and above the cone $\phi = \pi/3$.

Exercise 7. A solid Q is bounded by the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 2$. The density at $P(x, y, z)$ is directly proportional to the square of the distance from the origin to P. Find its mass. (Swok Sec 17.8 Ex 5)

Class Exercise 9. Use spherical coordinates. (#28-34 even)
(a) Find the average distance from a point in a ball of radius a to its center.
(b) Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2+y^2}$.
(c) Let H be a solid hemisphere of radius a whose density at any point is proportional to its distance from the center of the base.
(i) Find the mass of H.
(ii) Find the center of mass of H.
(iii) Find the moment of inertia of H about its axis.
(d) Find the mass and center of mass of a solid hemisphere of radius a if the density at any point is proportional to its distance from the base.

Class Exercise 10. Find the volume of the smaller wedge cut from a sphere of radius a by two planes that intersect along a diameter at an angle of $\pi/6$. (#36)

Class Exercise 11. Evaluate the integral by changing to spherical coordinates:

$$\int_{-a}^{a} \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2z + y^2z + z^3)\,dz\,dx\,dy. \text{ (\#40)}$$

Homework: 1-13 ODD, 17, 19, 23, 25, 27, 31(a), 43