Section 16.1

Definition: Let D be a set in $\mathbb{R}^2$ (a plane region). A **vector field on $\mathbb{R}^2$** is a function $\overrightarrow{F}$ that assigns to each point (x, y) in D a two-dimensional vector $\overrightarrow{F}(x, y)$.

Exercise 1. Describe the vector field $\overrightarrow{F}$ if $\overrightarrow{F}(x, y) = -y\ \overrightarrow{i} + x\ \overrightarrow{j}$. (Swok Sec 18.1 Ex 1)

Definition: Let E be a subset of $\mathbb{R}^3$. A **vector field on $\mathbb{R}^3$** is a function $\overrightarrow{F}$ that assigns to each point (x, y, z) in E a three-dimensional vector $\overrightarrow{F}(x, y, z)$.

Exercise 2. **Vector fields** in $\mathbb{R}^3$. Sketch and discuss the following vector fields.
(Briggs Sec 14.1 Ex 3)
(a) $\overrightarrow{F}(x, y, z) = < x,\ y,\ e^{-z} >$, for $z \geq 0$
(b) $\overrightarrow{F}(x, y, z) = < 0,\ 0,\ 1 - x^2 - y^2 >$, for $x^2 + y^2 \leq 1$

Class Exercise 1. Sketch the vector field $\overrightarrow{F}$ by drawing a diagram. (#2-10 even)
(a) $\overrightarrow{F}(x, y) = \frac{1}{2}x\overrightarrow{i} + y\overrightarrow{j}$ (b) $\overrightarrow{F}(x, y) = y\overrightarrow{i} + (x + y)\overrightarrow{j}$
(c) $\overrightarrow{F}(x, y) = \frac{y\overrightarrow{i} - x\overrightarrow{j}}{\sqrt{x^2+y^2}}$ (d) $\overrightarrow{F}(x, y, z) = -y\overrightarrow{k}$ (e) $\overrightarrow{F}(x, y, z) = \overrightarrow{j} - \overrightarrow{i}$

Definition: If f is a scalar function of two variables, recall that its gradient ∇f is defined by

$$\nabla f(x, y) = f_x(x, y)\ \overrightarrow{i} + f_y(x, y)\ \overrightarrow{j}.$$

Therefore, ∇f is really a vector field on $\mathbb{R}^2$ and is called a **gradient vector field**.

Exercise 3. Sketch and interpret the gradient field associated with the temperature function $T = 200 - x^2 - y^2$ on the circular plate $R = \{\ (x, y) : x^2 + y^2 \leq 25\ \}$. (Briggs Sec 14.1 Ex 4a)

Exercise 4. Sketch and interpret the gradient field associated with the velocity potential $\phi = \tan^{-1}(y/x)$. (Briggs Sec 14.1 Ex 4b)

Class Exercise 2. Find the gradient vector field of f. (#22,24)
(a) $f(x, y) = \tan(3x - 4y)$
(b) $f(x, y, z) = x\ \ln(y - 2z)$

Class Exercise 3. Find the gradient vector field ∇f of $f(x, y) = \sqrt{x^2 + y^2}$ and sketch it. (#26)

Definition: A vector field $\overrightarrow{F}$ is called a **conservative vector field** if it is the gradient of some scalar function, that is, if there exists a function f such that $\overrightarrow{F} = \nabla f$.

Definition: In the above situation, f is called a **potential function** for $\overrightarrow{F}$.

Homework: 1, 5-17 ODD, 25-33 ODD