Section 16.6

Definition: We can describe a surface by a vector function $\vec{r}(u,v)$ of two parameters u and v. We suppose that

$$\vec{r}(u,v) = x(u,v)\vec{i} + y(u,v)\vec{j} + z(u,v)\vec{k}$$

is a vector-valued function defined on a region D in the uv-plane. If $\vec{r}_u \; X \; \vec{r}_v$ is not $\vec{0}$, then the surface S is called **smooth**.

Exercise 1. Identify and sketch the surface with equation
$\vec{r}(u,v) = 2 \cos u \; \vec{i} + v \; \vec{j} + 2 \sin u \; \vec{k}$ (Stew Sec 16.6 Ex 1)

Class Exercise 1. Identify the surface with the given vector equation. (#4,6)
(a) $\vec{r}(u,v) = 2 \sin u \; \vec{i} + 3 \cos u \; \vec{j} + v \; \vec{k}$, $0 \le v \le 2$ (b) $\vec{r}(s,t) = < s \sin 2t, \, s^2, \, s \cos 2t >$

Exercise 2. Find a parametrization of the cone $z = \sqrt{x^2+y^2}$, $0 \le z \le 1$. (Hass Sec 16.5 Ex 1)

Exercise 3. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$. (Hass Sec 16.5 Ex 2)

Exercise 4. Find a parametrization of the cylinder $x^2 + (y-3)^2 = 9$, $0 \le z \le 5$. (Hass Sec 16.5 Ex 3)

Class Exercise 2. Find a parametric representation of the surface (#22-26 even).
(a) The part of the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ that lies to the left of the xz-plane
(b) The part of the sphere $x^2 + y^2 + z^2 = 16$ that lies between the planes $z = -2$ and $z = 2$
(c) The part of the plane $z = x + 3$ that lies inside the cylinder $x^2 + y^2 = 1$

Definition: For a smooth surface, the **tangent plane** is the plane that contains the tangent vectors $\vec{r}_u$ and $\vec{r}_v$ and the vector $\vec{r}_u \; X \; \vec{r}_v$ is a normal vector to the tangent plane.

Exercise 5. Find the tangent plane to the surface with parametric equations $x = u^2$, $y = v^2$, $z = u + 2v$ at the point (1,1,3). (Stew Sec 16.6 Ex 9)

Class Exercise 3. Find an equation of the tangent plane to the given parametric surface at the specified point. (#34,36)
(a) $x = u^2 + 1$, $y = v^3 + 1$, $z = u + v$; $(5, 2, 3)$
(b) $\vec{r}(u,v) = \sin u \; \vec{i} + \cos u \sin v \; \vec{j} + \sin v \; \vec{k}$; $u = \pi/6$, $v = \pi/6$

Definition: If a smooth parametric surface S is given by the equation

$$\vec{r}(u,v) = x(u,v)\vec{i} + y(u,v)\vec{j} + z(u,v)\vec{k} \qquad (u,v) \in D$$

and S is covered just once as (u,v) ranges throughout the parameter domain D, then the **surface area** of S is

$$A(S) = \iint_D | \vec{r}_u \; X \; \vec{r}_v | \, dA,$$

where $\vec{r}_u = \frac{\partial x}{\partial u}\vec{i} + \frac{\partial y}{\partial u}\vec{j} + \frac{\partial z}{\partial u}\vec{k}$ and $\vec{r}_v = \frac{\partial x}{\partial v}\vec{i} + \frac{\partial y}{\partial v}\vec{j} + \frac{\partial z}{\partial v}\vec{k}$

Exercise 6. Find the surface area of the (a) cone $z = \sqrt{x^2+y^2}$, $0 \le z \le 1$ and (b) a sphere of radius a. (Hass Sec 16.5 Ex 4, 5)

Class Exercise 4. Find the area of the surface. (#40-50 even)
(a) The part of the plane with vector equation $\vec{r}(u,v) = < u + v, \, 2 - 3u, \, 1 + u - v >$ that is given by $0 \le u \le 2$, $-1 \le v \le 1$.
(b) The part of the cone $z = \sqrt{x^2+y^2}$ that lies between the plane $y = x$ and the cylinder $y = x^2$.
(c) The part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices (0,0), (0,1), and (2,1).
(d) The part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$
(e) The helicoid (or spiral ramp) with vector equation $\vec{r}(u,v) = u \cos v \; \vec{i} + u \sin v \; \vec{j} + v\vec{k}$, $0 \le u \le 1$, $0 \le v \le \pi$.
(f) The part of the sphere $x^2 + y^2 + z^2 = b^2$ that lies inside the cylinder $x^2 + y^2 = a^2$, where $0 < a < b$.

Homework: 1, 5, 21, 25, 33, 35, 39-49 ODD