Section 16.7

The surface integral of f over the surface S is

$$\iint_S f(x,y,z)\ dS = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(P_{ij}^*) \triangle S_{ij}$$

Formula:

$$\iint_S f(x,y,z)\ dS = \iint_D f(\overrightarrow{r}(u,v))\ |\ \overrightarrow{r}_u\ X\ \overrightarrow{r}_v\ |\ dA$$

Formula

$$\iint_S f(x,y,z)\ dS = \iint_D f(x,y,g(x,y))\ \sqrt{(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 + 1}\ dA$$

Exercise 1. Evaluate $\iint_S x^2 z\ dS$ if S is the portion of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 4$. (Swok Sec 18.5 Ex 1)

Exercise 2. Evaluate $\iint_S (xz/y)\ dS$ if S is the portion of the cylinder $x = y^2$ that lies in the first octant between the planes $z = 0$, $z = 5$, $y = 1$, and $y = 4$. (Swok Sec 18.5 Ex 2)

Exercise 3. Evaluate $\iint_S (z+y)\ dS$ if S is the part of the graph of $z = \sqrt{1-x^2}$ in the first octant between the xz-plane and the plane $y = 3$. (Swok Sec 18.5 Ex 3)

Class Exercise 1. Evaluate the surface integral. (#6-20 even)
(a) $\iint_S xyz\ dS$, S is the cone with parametric equations $x = u \cos v$, $y = u \sin v$, $z = u$, $0 \le u \le 1$, $0 \le v \le \pi/2$
(b) $\iint_S (x^2 + y^2)\ dS$, S is the surface with vector equation:

$$\overrightarrow{r}(u,v) = <2uv,\ u^2 - v^2,\ u^2 + v^2>,\ u^2 + v^2 \le 1$$

(c) $\iint_S xz\ dS$, S is the part of the plane $2x + 2y + z = 4$ that lies in the first octant
(d) $\iint_S y\ dS$, S is the surface $z = \frac{2}{3}(x^{3/2} + y^{3/2})$, $0 \le x \le 1$, $0 \le y \le 1$
(e) $\iint_S z\ dS$, S is the surface $x = y + 2z^2$, $0 \le y \le 1$, $0 \le z \le 1$
(f) $\iint_S y^2\ dS$, S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane
(g) $\iint_S xz\ dS$, S is the boundary of the region enclosed by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$ and $x + y = 5$
(h) $\iint_S (x^2 + y^2 + z^2)\ dS$, S is the part of the surface of the cylinder $x^2 + y^2 = 9$ between the planes $z = 0$ and $z = 2$, together with its top and bottom disks.

Definition: If it is possible to choose a unit normal vector $\overrightarrow{n}$ at every such point (x, y, z) so that $\overrightarrow{n}$ varies continuously over S, then S is called an **oriented surface** and the given choice of $\overrightarrow{n}$ provides S with an **orientation**.

Definition: For a **closed surface**, that is, a surface that is the boundary of a solid region E, the convention is that the **positive orientation** is the one for which the normal vectors point outward from E.

Definition: If $\overrightarrow{F}$ is a continuous vector field defined on an oriented surface S with unit normal vector $\overrightarrow{n}$, then the **surface integral of $\overrightarrow{F}$ over S** is

$$\iint_S \overrightarrow{F} \cdot d\overrightarrow{S} = \iint_S \overrightarrow{F} \cdot \overrightarrow{n}\ dS.$$

This integral is called the **flux of $\overrightarrow{F}$** across S.

Formula:

$$\iint_S \overrightarrow{F} \cdot d\overrightarrow{S} = \iint_D \overrightarrow{F} \cdot (\overrightarrow{r}_u\ X\ \overrightarrow{r}_v)\ dA.$$

Formula:

$$\iint_S \overrightarrow{F} \cdot d\overrightarrow{S} = \iint_D (-P\tfrac{\partial g}{\partial x} - Q\tfrac{\partial g}{\partial y} + R)\ dA.$$

Exercise 4. Consider the radial vector field $\overrightarrow{F} = < x,\ y,\ z >$. Find the upward flux of the field across the hemisphere $x^2 + y^2 + z^2 = 1$, for $z \geq 0$. (Briggs Sec 14.8 Ex 8a)

Exercise 5. Consider the radial vector field $\overrightarrow{F} = < x,\ y,\ z >$. Find the upward flux of the field across the paraboloid $z = 1 - x^2 - y^2$, for $z \geq 0$. (Briggs Sec 14.8 Ex 8b)

Exercise 6. Let S be the part of the graph of $z = 9 - x^2 - y^2$ with $z \geq 0$.
If $\overrightarrow{F}(x, y, z) = 3x\overrightarrow{i} + 3y\overrightarrow{j} + z\overrightarrow{k}$, find the flux of $\overrightarrow{F}$ through S. (Swok Sec 18.5 Ex 5)

Class Exercise 2. Evaluate the surface integral $\iint_S \overrightarrow{F} \cdot d\overrightarrow{S}$ for the given vector field $\overrightarrow{F}$ and the oriented surface S. In other words, find the flux of $\overrightarrow{F}$ across S. For closed surfaces, use the positive (outward) orientation. (#24-32 even)
(a) $\overrightarrow{F}(x, y, z) = -x\overrightarrow{i} + -y\overrightarrow{j} + z^3\overrightarrow{k}$, S is the part of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 3$ with downward orientation
(b) $\overrightarrow{F}(x, y, z) = xz\overrightarrow{i} + x\overrightarrow{j} + y\overrightarrow{k}$, S is the hemisphere $x^2 + y^2 + z^2 = 25$, $y \geq 0$, oriented in the direction of the positive y-axis
(c) $\overrightarrow{F}(x, y, z) = xy\overrightarrow{i} + 4x^2\overrightarrow{j} + yz\overrightarrow{k}$, S is the surface $z = xe^y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$, with upward orientation
(d) $\overrightarrow{F}(x, y, z) = x\overrightarrow{i} + y\overrightarrow{j} + 5\overrightarrow{k}$, S is the boundary of the region enclosed by the cylinder $x^2 + z^2 = 1$ and the planes $y = 0$ and $x + y = 2$
(e) $\overrightarrow{F}(x, y, z) = y\overrightarrow{i} + (z - y)\overrightarrow{j} + x\overrightarrow{k}$, S is the surface of the tetrahedron with vertices (0,0,0), (1,0,0), (0,1,0), (0,0,1)

Homework: 1, 3, 5-17 (every 4th), 19-31 (every 4th)