Section 16.8

Stokes' Theorem: Let S be an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation. Let $\overrightarrow{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ that contains S. Then,

$$\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = \iint_S \text{curl } \overrightarrow{F} \cdot d\overrightarrow{S}$$

Exercise 1. Evaluate the integral $\iint_S (\nabla \ X \ \overrightarrow{F}) \cdot \overrightarrow{n} \ dS$, where $\overrightarrow{F} = -xz \ \overrightarrow{i} + yz \ \overrightarrow{j} + xye^z \ \overrightarrow{k}$ and S is the cap of the paraboloid $z = 5 - x^2 - y^2$ above the plane $z = 3$. Assume $\overrightarrow{n}$ points in the upward direction on S. (Briggs Sec 14.7 Ex 3)

Exercise 2. Let the surface S be the elliptical paraboloid $z = x^2 + 4y^2$ lying beneath the plane $z = 1$. We define the orientation of S by taking the inner normal vector $\overrightarrow{n}$ to the surface, which is the normal vector having a positive $\overrightarrow{k}$-component. Find the flux of $\nabla \ X \ \overrightarrow{F}$ across S in the direction $\overrightarrow{n}$ for the vector field $\overrightarrow{F} = y\overrightarrow{i} - xz \ \overrightarrow{j} + xz^2\overrightarrow{k}$. (Hass Sec 16.7 Ex 10)

Class Exercise 1. Use Stokes' Theorem to evaluate $\iint_S \text{curl } \overrightarrow{F} \cdot d\overrightarrow{S}$. (#2,4,6)

(a) $\overrightarrow{F}(x, y, z) = 2y \cos z \ \overrightarrow{i} + e^x \sin z \ \overrightarrow{j} + xe^y\overrightarrow{k}$, S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$, oriented upward

(b) $\overrightarrow{F}(x, y, z) = \tan^{-1}(x^2yz^2)\overrightarrow{i} + x^2y\overrightarrow{j} + x^2z^2\overrightarrow{k}$, S is the cone $x = \sqrt{y^2 + z^2}$, $0 \leq x \leq 2$, oriented in the direction of the positive x-axis

(c) $\overrightarrow{F}(x, y, z) = e^{xy}\overrightarrow{i} + e^{xz}\overrightarrow{j} + x^2z\overrightarrow{k}$, S is the half of the ellipsoid $4x^2 + y^2 + 4z^2 = 4$ that lies to the right of the xz-plane, oriented in the direction of the positive y-axis

Exercise 3. Evaluate the line integral $\oint \overrightarrow{F} \cdot d\overrightarrow{r}$, where $\overrightarrow{F} = z\overrightarrow{i} - z\overrightarrow{j} + (x^2 - y^2)\overrightarrow{k}$ and C consists of the three line segments that bound the plane $z = 8 - 4x - 2y$ in the first octant. (Briggs Sec 14.7 Ex 2)

Exercise 4. Evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$, where $\overrightarrow{F}(x, y, z) = -y^2\overrightarrow{i} + x\overrightarrow{j} + z^2\overrightarrow{k}$ and C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$. (Stew Sec 16.8 Ex 1)

Class Exercise 2. Use Stokes' Theorem to evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$. In each case C is oriented counterclockwise as viewed from above. (#8,10)

(a) $\overrightarrow{F}(x, y, z) = \overrightarrow{i} + (x + yz)\overrightarrow{j} + (xy - \sqrt{z})\overrightarrow{k}$, C is the boundary of the part of the plane $3x + 2y + z = 1$ in the first octant

(b) $\overrightarrow{F}(x, y, z) = xy\overrightarrow{i} + 2z\overrightarrow{j} + 3y\overrightarrow{k}$, C is the curve of intersection of the plane $x + z = 5$ and the cylinder $x^2 + y^2 = 9$

Exercise 5. Let S be the part of the paraboloid $z = 9 - x^2 - y^2$ with $z \geq 0$, and let C be the trace of S on the xy-plane. Verify Stokes' Theorem for the vector field $\overrightarrow{F} = 3z\overrightarrow{i} + 4x\overrightarrow{j} + 2y\overrightarrow{k}$. (Swok Sec 18.7 Ex 1)

Class Exercise 3. Verify that Stokes' Theorem is true for the given vector field $\overrightarrow{F}$ and surface S. $\overrightarrow{F}(x, y, z) = -2yz\overrightarrow{i} + y\overrightarrow{j} + 3x\overrightarrow{k}$, S is the part of the paraboloid $z = 5 - x^2 - y^2$ that lies above the plane $z = 1$, oriented upward. (#14)

Class Exercise 4. Let C be a simple closed smooth curve that lies in the plane $x + y + z = 1$. Show that the line integral

$$\int_C z \ dx - 2x \ dy + 3y \ dz$$

depends only on the area of the region enclosed by C and not on the shape of C or its location in the plane. (#16)

Class Exercise 5. Evaluate

$$\int_C (y + \sin x \ ) \ dx + (z^2 + \cos y) \ dy + x^3 \ dz$$

where C is the curve $\overrightarrow{r}(t) = < \sin t, \cos t, \sin 2t >$, $0 \leq t \leq 2\pi$. (#18)

Homework: 3-9 ODD, 17-21 ODD