Section 5.5

The Substitution Rule: If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x)\ dx = \int f(u)\ du.$$

The trick is to use up the whole integrand (and dx) with some combination of a function and its derivative. The du (or the derivative) must exist as a multiple. The u (or the function) can be nested as part of a composition, the denominator, or . . .

Exercise 1. Evaluate the integral:
(a) $\int \frac{5x}{\sqrt{x^2+3}}\ dx$. (SD 6.2 #1)
(b) $\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\ dx$. (SD 6.2 #2)
(c) $\int \frac{x^2-1}{(x^3-3x+1)^6}\ dx$. (SD 6.2 #3)
(d) $\int \frac{3}{\sqrt{4-5t}}\ dt$. (SD 6.2 #4)

Class Exercise 1. Evaluate the integral:
(a) $\int \cos^3 5x \cdot \sin 5x\ dx$. (SD 6.2 #5)
(b) $\int (s^2+1)^2\ ds$. (SD 6.2 #6)
(c) $\int \frac{1}{x(\log_4 x)^2}\ dx$. (SD 6.2 #7)
(d) $\int \frac{1}{\sqrt{9-x^2}}\ dx$. (SD 6.2 #8)
(e) $\int \frac{\sin 4x}{\cos 2x}\ dx$. (SD 6.2 #9)
(f) $\int \frac{\sin y}{1+\cos^2 y}\ dy$. (SD 6.2 #10)

Substitution Rule for Definite Integrals: If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x)) \cdot g'(x)\ dx = \int_{g(a)}^{g(b)} f(u)\ du.$$

Exercise 2. Evaluate the integral:
(a) $\int_{-\pi/4}^{\pi/3} \tan x$ dx. (SD 6.2 #11)
(b) $\int_{\pi/2}^{5\pi/6} \sqrt{\csc^5\theta} \cot\theta$ dθ. (SD 6.2 #12)
(c) $\int_{-\pi/6}^{\pi/6} (x + \sin 5x)$ dx. (SD 6.2 #13)

Integrals of Symmetric Functions: Suppose f is continuous on $[-a, a]$.
(a) If f is even $[f(-x) = f(x)]$, then $\int_{-a}^{a} f(x)\ dx = 2\int_0^a f(x)\ dx$.
(b) If f is odd $[f(-x) = -f(x)]$, then $\int_{-a}^{a} f(x)\ dx = 0$.

Exercise 3. Evaluate the integral: $\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6}\ dx$.

Exercise 4. Evaluate the integral: $\int_{-2}^{2} x^6 + 1\ dx$. (Example 11)

Class Exercise 2. Evaluate the integral:
(a) $\int_0^1 \frac{1}{(3-2x)^2}\ dx$. (SD #14)
(b) $\int_{-1}^{2} 4^x\sqrt{1+4^x}\ dx$. (SD #15)
(c) $\int_0^{\frac{1}{2}} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\ dx$. (SD #16)
(d) $\int_1^3 \frac{e^x+1}{e^x}\ dx$. (SD #17)
(e) $\int_0^4 x\sqrt{16-3x}\ dx$. (SD 6.2 #18)
(f) $\int_{-1}^{1} \frac{x+x^3+x^5}{1+x^2+x^4}\ dx$. (SD #19)

Homework: 3, 9, 13, 19, 23, 27, 35, 41-89 (every 4th)

Section 6.1

<u>Area Between Curves</u>: If f and g are continuous with $f(x) \geq g(x)$ throughout $[a, b]$, then the area between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $[f - g]$ from a to b,

$$A = \int_a^b [f(x) - g(x)]\ dx.$$

Exercise 5. (a) Find the area between $y = \sec^2 x$ and $y = \sin x$ from $x = 0$ to $x = \pi/4$.
(b) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
(c) Find the area of the region R in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

Class Exercise 3. Find the area enclosed by the given curves.
(a) $y = \sqrt[3]{2x}$ and $y = \frac{1}{2}x$
(b) $y = \sqrt{x}$, $y = \frac{1}{3}x$, $0 \leq x \leq 16$
(c) $y = \sec^2 x$, $y = 8 \cos x$, $-\pi/3 \leq x \leq \pi/3$

If y is the independent variable, use $\int_c^d (f(y) - g(y))\ dy$ where the area starts at $y = c$, stops at $y = d$, $f(y)$ is on the right and $g(y)$ is on the left (think right minus left).

Exercise 6. Find the area in the last problem by integrating with respect to y.

Exercise 7. Set up an integral to find the area of the regions enclosed by the lines and curves.
(a) $y = \frac{1}{2}x + \frac{1}{2}$, $y = \frac{1}{x^2}$, and $x = 3$.
(b) $x = 2y$ and $x + y^2 = 8$.
(c) $y = x + 6$, $y = -\frac{1}{2}x$, and $y = x^3$.

Class Exercise 4. Set up an integral to find the area of the regions enclosed by the lines and curves.
(a) $y = \cos x$, $y = \sin x$, and $x = 0$.
(b) $y = 4 \sin^3 x \cos x$ and $y = x(x - \frac{\pi}{2})$.
(c) $y = 8$, $x = y^{2/3}$, and $x - y = 2$.
(d) $x = y^2$ and $x = 2y^2$ - 4.

Homework: 1-17 (every 4th), 19, 21, 27, 31, 35