Section 6.3

<u>Definition</u>: Let f be continuous and suppose $f(x) \geq 0$ on $[a, b]$, where $0 \leq a \leq b$. Let R be the region under the graph of f from a to b. The volume V of the solid of revolution generated by revolving R about the y-axis is

$$V = \int_a^b 2\pi\, x\, f(x)\, dx.$$

Here are the guidelines for finding the volume of a solid of revolution using cylindrical shells.
1. Sketch the region R to be revolved, and label the boundaries. Show a typical vertical rectangle of width dx or a horizontal rectangle of width dy.
2. Express the average radius of the shell in terms of x or y, depending on whether its thickness is dx or dy. Remember that x represents a distance from the y-axis to a vertical rectangle, and y represents a distance from the x-axis to a horizontal rectangle.
3. Express the altitude of the shell in terms of x or y, depending on whether its thickness is dx or dy.
4. Use the formula above to find a formula for the volume of the shell.
5. Apply the limit of sums operator $\int_a^b$ or $\int_c^d$ to the expression in guideline 4 and evaluate the integral.

Exercise 1. The region bounded by the graph of $y = 2x - x^2$ and the x-axis is revolved about the y-axis. Find the volume of the resulting solid.

Exercise 2. The region bounded by the graphs of $y = x^2$ and $y = x + 2$ is revolved about the line $x = 3$. Set up the integral for the volume of the resulting solid.

Exercise 3. The region in the first quadrant bounded by the graphs of the equation $x = 2y^3 - y^4$ and the y-axis is revolved about the x-axis. Set up the integral for the volume of the resulting solid.

Class Exercise 1. Use the method of cylindrical shells to set up an integral to find the volume generated by rotating the region bounded by the given curves about the specified axis.
(a) $y = x + 1$, $y = (x - 1)^2$; y-axis.
(b) $y = \frac{1}{2}x + \frac{1}{2}$, $y = \frac{1}{x^2}$, $x = 3$; y-axis.
(c) $y = \frac{1}{2}x$, $y = 0$, $x = 4$; x-axis.
(d) $y = 2x$, $y = \frac{1}{8}x^3$, $x > 0$; y-axis.
(e) $y = -x^3$, $y = -\sqrt{x}$; x-axis.
(f) $y = \cos x$, $y = \sin x$, $x = 0$; y-axis.
(g) $x = y^{2/3}$, x - $y = 2$, $y = 8$, $y = 2$; x-axis.

Here is the procedure for setting up a volume integral for a solid of revolution:
(1) Determine the independent variable (x or y).
(2) Determine the axis of rotation (x or y).
(3) If the variables match, use washers. If the variables don't match, use shells.
(4) Set up all components using top - bottom or right - left.

Exercise 4. Set up the integral to find the volume generated by rotating the region bounded by the given curves about the specified line.
(a) $y = x + 1$, $y = (x - 1)^2$, $x = 3$.
(b) $x = 2y$, $x + y^2 = 8$, $y = 2$.
(c) $y = x + 6$, $y = -\frac{1}{2}x$, $x = 1$; y = -1.

Class Exercise 2. Set up the integral to find the volume generated by rotating the region bounded by the given curves about the specified line.
(a) $x = 2y^2 - 4$, $x = y^2$; $x = 4$.
(b) $y = \sin x$ and $y = \cos x$, on the interval $[\frac{\pi}{4}, \frac{5\pi}{4}]$; $x = \frac{\pi}{4}$.
(c) $y = \frac{1}{2}x^2 + 2$, $y = 0$, x = -1, $x = 2$; $y = 5$.
(d) $x = y^{2/3}$, $y = 8$, $x - y = 2$; y = -2.
(e) $y = 2x$, $y = \frac{1}{8}x^3$; $y = 8$.

Homework: 3-59 (every 4th)