Section 6.4

The concept of force may be considered as a push or a pull on an object. For example, a force is needed to push or pull furniture across a floor, to lift an object off the ground, to stretch or compress a spring, or to move a charged particle through an electromagnetic field.

If an object weighs 10 pounds, then by definition the force required to lift it (or hold it off the ground) is 10 pounds. A force of this type is a constant force, since its magnitude does not change while it is applied to the object.

The concept of work is used when a force acts through a distance. The following definition covers the simplest case, in which the object moves along a line in the same direction as the applied force.

If a constant force F acts on an object, moving it a distance d in the direction of the force, the work W done is

$$W = Fd.$$

The following table lists units of force and work in the old British system and the International System. In SI units, 1 Newton is the force required to impart an acceleration of 1 m/sec^2 to a mass of 1 kilogram.

System	Unit of force	Unit of distance	Unit of work
old British	pound (lb)	foot (ft), inch (in.)	foot-pound (ft-lb)
International (SI)	Newton (N)	meter (m)	Newton-meter (N-m)

A Newton-meter is also called a *joule* (J). It can be shown that

$$1 \text{ N} \approx 0.225 \text{ lb and } 1 \text{ N-m} \approx 0.74 \text{ ft-lb}.$$

Exercise 1. Find the work done in pushing an automobile a distance of 20 feet along a level road while exerting a constant force of 90 pounds.

Definition: If $f(x)$ is the force at x and if f is continuous on $[a, b]$, then the work W done in moving an object along the x-axis from $x = a$ to $x = b$ is

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*) \triangle x_k = \int_a^b f(x)\, dx.$$

Hooke's Law: The force $f(x)$ required to stretch a spring x units beyond its natural length is given by $f(x) = kx$, where k is a constant called the spring constant.

Exercise 2. A force of 9 pounds is required to stretch a spring from its natural length of 6 inches to a length of 8 inches. Find the work done in stretching the spring
(a) from its natural length to a length of 10 inches
(b) from a length of 7 inches to a length of 9 inches

Class Exercise 1. A spring has a natural length of 10 in. An 800-lb force stretches the spring to 14 in.
(a) Find the force constant.
(b) How much work is done in stretching the spring from 10 in. to 12 in.?
(c) How far beyond its natural length will a 1600-lb force stretch the spring?

Class Exercise 2. It takes a force of 21,714 lb to compress a coil spring assembly on a New York City Transit Authority subway car from its free height of 8 in. to its fully compressed height of 5 in.
(a) What is the assembly's force constant?
(b) How much work does it take to compress the assembly the first half inch? the second half inch? Answer to the nearest inch-pound.

Exercise 3. A uniform cable 30 feet long and weighing 60 pounds hangs vertically from a pulley system at the top of a building. A steel beam that weighs 500 pounds is attached to the end of the cable; find the work required to pull it to the top.

Exercise 4. A right circular conical tank of altitude 20 feet and radius of base 5 feet has its vertex at ground level and axis vertical. If the tank is full of water weighing 62.5 lb/ft^3, find the work done in pumping all the water over the top of the tank.

Class Exercise 3. A leaky bucket weighs 22 newtons (N) empty. It is lifted from the ground at a constant rate to a point 20 m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70 N (approximately 7.1 liters) of water, but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done
(a) lifting the bucket alone;
(b) lifting the water alone;
(c) lifting the rope alone;
(d) lifting the bucket, water, and rope together.

Class Exercise 4. A bucket is to be lifted 12 feet. The bucket weighs 2 lbs, and it is initially filled with 4 lbs. of water. However, water is leaking from the bucket at a constant rate, so that only 1 lb. of water remains after its trip of 12 feet. Find the work done in lifting the bucket and water.

Class Exercise 5. The conical tank in the figure on the board is filled to within 2 feet of the top with olive oil weighing 57 lb/ft^3. How much work is does it take to pump the oil to the rim of the tank?

Homework: 1-11 ODD, 13, 19