Section 8.1

In this section, we will be using integrals to find the length of the graph of a function.

Mean Value Theorem: If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

Definition: A function f is **smooth** on an interval if it has a derivative f' that is continuous throughout the interval. (Intuitively, this means that a small change in x produces a small change in the slope $f'(x)$ of the tangent line to the graph of f.)

Definition: Let f be smooth on $[a, b]$. The **arc length of the graph** of f from $A(a, f(a))$ to $B(b, f(b))$ is

$$L_a^b = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Exercise 1. Find the length of the curve, $y = \frac{4\sqrt{2}}{3}x^{3/2}$ - 1, $0 \le x \le 1$.

Class Exercise 1. Find the lengths of the curves.
(a) $y = (1/3)(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.
(b) $y = x^{3/2}$ from $x = 0$ to $x = 4$.

Exercise 2. Find the length of the graph of

$$f(x) = \frac{x^3}{12} + \frac{1}{x},\ 1 \le x \le 4.$$

Exercise 3. If $f(x) = 3x^{2/3} - 10$, find the length of the graph of f from the point $A(8, 2)$ to $B(27, 17)$.

Class Exercise 2. Find the length of the curves.
(a) $y = \frac{1}{2}(e^x + e^{-x})$, $0 \le x \le 2$.
(b) $y = x^3/3 + x^2 + x + 1/(4x + 4)$, $0 \le x \le 2$.
(c) $y = \int_{-2}^{x} \sqrt{3t^4 - 1}\, dt$, $-2 \le x \le -1$

Definition: If g' is continuous on $[c, d]$, the length of the curve $x = g(y)$ from $A = (g(c), c)$ to $B = (g(d), d)$ is

$$L = \int_c^d \sqrt{1 + (dx/dy)^2}\, dy = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy.$$

Exercise 4. Set up an integral for finding the arc length of the graph of the equation $y^3 - y - x = 0$ from $A(0, -1)$ to $B(6, 2)$.

Class Exercise 3. Find the lengths of the curves.
(a) $x = (y^3/3) + 1/(4y)$ from $y = 1$ to $y = 3$.
(b) $x = (y^4/4) + 1/(8y^2)$ from $y = 1$ to $y = 2$.
(c) $x = y^3/6 + 1/(2y)$ from $y = 1$ to $y = 2$.
(d) $x = \int_0^y \sqrt{\sec^4 t - 1}\, dt$, $-\pi/4 \le y \le \pi/4$.

Homework: 1-9 ODD, 13, 17-23 ODD