Section 8.2

Definition: If f is smooth and $f(x) \geq 0$ on $[a, b]$, then the **area** S of the surface generated by revolving the graph of f about the x-axis is

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} \ dx.$$

Exercise 1. The graph of $y = \sqrt{x}$ from (1,1) to (4,2) is revolved about the x-axis. Find the area of the resulting surface.

Class Exercise 1. The given curve is rotated about the x-axis. Find the area of the resulting surface.
(a) $9x = y^2 + 18$, $2 \leq x \leq 6$.
(b) $y = \sqrt{1 + e^x}$, $0 \leq x \leq 1$.
(c) $y = \frac{x^3}{6} + \frac{1}{2x}$, $\frac{1}{2} \leq x \leq 1$
(d) $y = 2\sqrt{x}$, $1 \leq x \leq 2$

Definition. If $x = g(y)$ and g is smooth and nonnegative on $[c, d]$, then the area S of the surface generated by revolving the graph of g about the y-axis is

$$S = \int_c^d 2\pi \ g(y) \ \sqrt{1 + [g'(y)]^2} \ dy = \int_a^b 2\pi \ x \ \sqrt{1 + [f'(x)]^2} \ dx.$$

Exercise 2. The given curve is rotated about the y-axis. Find the area of the resulting surface.

$$y = 1 - x^2, \ 0 \leq x \leq 1.$$

Class Exercise 2. The given curve is rotated about the y-axis. Find the area of the resulting surface.
(a) $x = \sqrt{y}$, $0 \leq y \leq 2$
(b) $x = y^3/3$, $0 \leq y \leq 1$
(c) $x = \sqrt{2y - 1}$, $(5/8) \leq y \leq 1$

Homework: 3-19 ODD