Section 8.3

Definition: The **pressure** p at depth h in a fluid is defined as the weight of fluid contained in a column that has a cross-sectional area of one square unit and an altitude h.

If a fluid has density ρ, then the pressure p at depth h is given by

$$p = \rho\ h.$$

Pascal's Principle: The pressure at a depth h in a fluid is the same in every direction.

Exercise 1. The ends of a water trough 8 feet long have the shape of isosceles trapezoids of lower base 4 feet, upper base 6 feet, and height 4 feet. Find the total force on one end if the trough is full of water. Assume that the density of water is 62.5 lb/ft^3.

Exercise 2. A cylindrical oil storage tank 6 feet in diameter and 10 feet long is lying on its side. If the tank is half full of oil that weighs 58 lb/ft^3, set up an integral for the force exerted by the oil on one end of the tank.

Class Exercise 1. A vertical plate is submerged (or partially submerged) in water and has the shape given by Mr. V in class. Explain how to approximate the hydrostatic force against one side of the plate by a Riemann sum. Then express the force as an integral and evaluate it.

Definition: Let S denote a system of point-masses m_1, m_2, ..., m_n located at x_1, x_2, ..., x_n on a coordinate line, and let $m = \sum m_k$ denote the total mass.

(i) The **moment of S about the origin** is $M_0 = \sum_{k=1}^{n} m_k x_k$.

(ii) The **center of mass** (or **center of gravity**) of S is given by $\bar{x} = \frac{M_0}{m}$.

Exercise 3. Three point-masses of 40, 60, and 100 kilograms are located at -2, 3, and 7, respectively, on an x-axis. Find the center of mass.

Class Exercise 2. Three point-masses of 40, 20, and 80 kilograms are located at -4, 1, and 2, respectively, on an x-axis. Find the center of mass.

Definition: Let S denote a system of point-masses m_1, m_2,......., m_n located at (x_1, y_1), (x_2, y_2),......, (x_n, y_n) in a coordinate plane, and let $m = \sum_{k=1}^{n} m_k$ denote the total mass.

(i) The **moment of S about the x-axis** is $M_x = \sum_{k=1}^{n} m_k y_k$.
(ii) The **moment of S about the y-axis** is $M_y = \sum_{k=1}^{n} m_k x_k$.
(iii) The **center of mass** (or **center of gravity**) of S is the point $(\bar{x}, \bar{y})$ such that

$$\bar{x} = \frac{M_y}{m},\ \bar{y} = \frac{M_x}{m}.$$

Exercise 4. Point-masses of 4, 8, 3, and 2 kilograms are located at (-2,3), (2,-6), (7,-3), and (5,1), respectively. Find M_x, M_y, and the center of mass of the system.

Class Exercise 3. The masses m_i are located at the points P_i. Find the moments M_x and M_y and the center of mass of the system.

$m_1 = 5$, $m_2 = 4$, $m_3 = 3$, $m_4 = 6$;

$P_1(-4, 2)$, $P_2(0, 5)$, $P_3(3, 2)$, $P_4(1, -2)$

Consider a flat plate (called a *lamina*) in the xy-plane with uniform density ρ that occupies a region R of the plate. Suppose that R is bounded by the curve $y = f(x)$.

The moment of a region R about the y-axis is:

$$M_y = \lim_{n\to\infty} \sum_{i=1}^{n} \rho\ \bar{x}_i \quad f(\bar{x}_i) \bigtriangleup x = \rho \int_a^b x f(x)\ dx.$$

The moment of a region R about the x-axis is:

$$M_x = \lim_{n\to\infty} \sum_{i=1}^{n} \rho \, \frac{1}{2}[f(\bar{x}_i)]^2 \triangle x = \rho \int_a^b \frac{1}{2} \, [f(x)]^2 \, dx.$$

The center of mass of the plate (or the centroid of R) is located at the point $(\bar{x}, \bar{y})$, where

$$\bar{x} = \frac{\int_a^b xf(x) \, dx}{\int_a^b f(x) \, dx} \qquad \bar{y} = \frac{\int_a^b \frac{1}{2} \, [f(x)]^2 \, dx}{\int_a^b f(x) dx}.$$

Exercise 5. Find the centroid of the semicircular region bounded by the x-axis on the graph of $y = \sqrt{a^2 - x^2}$ with $a > 0$.

Class Exercise 4. Find the centroid of the region bounded by the given curves.
(a) $y = \sqrt{x}$, $y = 0$, and $x = 4$
(b) $y = \sin x$, $y = 0$, $0 \le x \le \pi$
(c) $y = 2 - x^2$ and $y = x$
(d) $y = x^3$, $x + y = 2$, and $y = 0$

Homework: 1, 3, 7, 15, 21-37 ODD