Section 9.1

In this section, we learn about differential equations.

Definition: A **differential equation** is an equation that contains an unknown function and some of its derivatives.

Strictly speaking, a number of individuals in a population is a discontinuous function of time because it takes on only whole number values. However, one common way to model a population is with a differentiable function P growing at a rate proportional to the size of the population. Thus, for some constant k,

$$\frac{dP}{dt} = kP.$$

Notice that

$$\frac{dP/dt}{P} = k$$

is constant. The rate is called the relative growth rate.

Exercise 1. Show that the solution of the differential equation is $P = P_0 e^{kt}$, where P_0 is the size of the population at time $t = 0$.

In reality, the relative growth rate is positive, but decreases as the population increases due to environmental or economic factors. In other words there is a maximum population M, the **carrying capacity**, that the environment is capable of sustaining in the long run. If we assume the relative growth rate is proportional to 1 - (P/M) with positive proportionality constant k, then

$$\frac{dP/dt}{P} = k\ (1 - \frac{P}{M}) \text{ or } \frac{dP}{dt} = \frac{k}{M}P(M - P).$$

The solution to this **logistic differential equation** is called the **logistic growth model**. Notice that the rate of growth is proportional to both P *and* $(M - P)$, where M is the assumed maximum population. If P were to exceed M, the growth rate would be negative (as $k > 0$, $M > 0$) and the population would be decreasing.

Exercise 2. Show that every member of the family of functions $y = (\ln x + C)/x$ is a solution of the differential equation $x^2 y' + xy = 1$.

Exercise 3. Find a solution of the differential equation that satisfies the initial condition $y(1) = 2$.

Class Exercise 1. Find a solution of the differential equation that satisfies the initial condition $y(2) = 1$.

Class Exercise 2. Verify that $y = -t \cos t - t$ is a solution of the initial-value problem

$$t\frac{dy}{dt} = y + t^2 \sin t \qquad y(\pi) = 0.$$

Homework: 7, 15-23 ODD