Section 9.3

<u>Definition</u>: If y is a function of x and if n is a positive integer, then an equation that involves x, y, y', y'',......., $y^{(n)}$ is called an **<u>ordinary differential equation of order n</u>**.

<u>Example</u>: The differential equation $y' = 2x$ is a first-order differential equation.

<u>Example</u>: The differential equation $\frac{d^2y}{dx^2} + x^2(\frac{dy}{dx})^3 - 15y = 0$ is a second order differential equation.

<u>Example</u>: The differential equation $(y^{'''})^4 - x^2(y^{''})^5 + 4xy = xe^x$ is a third order differential equation.

<u>Definition</u>: A **<u>separable equation</u>** is a first-order differential equation in which the expression for dy/dx can be factored as a function of x times a function of y. In other words, it can be written in the form

$$\frac{dy}{dx} = g(x)f(y).$$

The name *separable* comes from the fact that the expression on the right side can be "separated" into a function of x and a function of y. Equivalently, if $f(y) \neq 0$, we could write

$$\frac{dy}{dx} = \frac{g(x)}{h(y)}.$$

Exercise 1. Given the differential equation $y' = 2x$,
(a) find the general solution and illustrate it graphically
(b) find the particular solution that satisfies the condition $y = 3$ if $x = 0$

Exercise 2. Solve the differential equation: $\frac{dy}{dx} = xe^{-y}$.

Class Exercise 1. Solve the differential equation.
(a) $(y^2 + xy^2)y' = 1$
(b) $\frac{dv}{ds} = \frac{s+1}{sv+s}$
(c) $\frac{dy}{d\theta} = \frac{e^y \sin^2\theta}{y\sec\theta}$

Exercise 3. Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{dy}{dx} = \frac{\ln x}{xy},\ y(1) = 2.$$

Class Exercise 2. Find the solution of the differential equation that satisfies the given initial condition.
(a) $y' = \frac{xy\sin x}{y+1}$, $y(0) = 1$
(b) $\frac{dP}{dt} = \sqrt{Pt}$, $P(1) = 2$
(c) $\frac{dL}{dt} = kL^2\ln t$, $L(1) =$ -1

<u>Definition</u>: An **<u>orthogonal trajectory</u>** of a family of curves is a curve that intersects each curve of the family orthogonally.

Exercise 4. Find the orthogonal trajectories of the family of ellipses $x^2 + 3y^2 = c$, and sketch several members of each family.

Class Exercise 3. Find the orthogonal trajectories of the family of curves.
(a) $y^2 = kx^3$
(b) $y = \frac{x}{1+kx}$

Homework: 1, 3-11 ODD, 13, 17, 39-55 (every 4th)